NEW YORK UNIVERSITY

College of Engineering
RESEARCH DIVISION
University Heights, New York 53, N. Y.

THE POWER SPECTRAL ANALYSIS OF CONCURRENT AIRPLANE AND TOWER MEASUREMENTS OF ATMOSPHERIC TURBULENCE

By
U. Oscar Lappe
and
Ben Davidson

Final Report Under Contract No. NOas 58-517-d

January 1960

The research reported in this document has been sponsored by the Bureau of Naval Weapons, Department of the Navy, Washington, D. C.

NEW YORK UNIVERSITY
COLLEGE OF ENGINEERING
RESEARCH DIVISION

THE POWER SPECTRAL ANALYSIS OF CONCURRENT AIRPLANE AND TOWER MEASUREMENTS OF ATMOSPHERIC TURBULENCE

By
U. Oscar Lappe
and
Ben Davidson

Final Report Under Contract No. NOas 58-517-d

The research reported in this document has been sponsored by the Bureau of Naval Weapons, Department of the Navy, Washington, D. C.

Department of Meteorology and Oceanography

January 1960

ABSTRACT

Under the present contract a power spectral analysis of concurrent airplane and tower data obtained during operation Tri-Tower was conducted. The objectives of this analysis were (1) determine the equivalence of the airplane and tower measurements and (2) investigate spectral behavior with respect to meteorological and terrain conditions.

Acknowledgment: The authors wish to express their deep appreciation to Dr. L. J. Tick and the staff of the Engineering Computation Laboratory for conducting the spectral computations of the tower data in this progam.

FOREWORD

This report presents an analysis of turbulence data collected by Cornell Aeronautical Laboratory under Air Force Project LOW BLOW (AF 18 (600) - 1550). The data was obtained during Operation Tri-Tower in April 1956. Assisting in this operation were meteorological groups from NYU, Brookhaven National Laboratories, and MIT (Round Hill Field Station). A change in Air Force policy in 1956-57 precluded the analysis of the data following the Tri-Tower operation.

In March 1957, the Structures Branch of the Airframe Design Division of the Bureau of Aeronautics, Navy Department, sponsored analysis of these data at CAL and NYU. Mr. Edward J. Griffin of the Airframe Division of the Bureau of Aeronautics was project administrator. The CAL contract (NOas 59-274-c) provided for the spectral density analysis of the airplane data, while the NYU contract (NOas 58-517-d) provided for a similar analysis of the tower data and for correlating the behavior of the airplane and tower measurements of turbulence for the conditions tested.

Mr. Charles B. Notess was project engineer of the CAL program and Mr. U. Oscar Lappe, who was responsible for the LOW BLOW project while at CAL, was project director of the NYU program. The CAL project was completed in early 1959,* while the NYU project was completed in the latter half of 1959. In January of 1959 a paper was prepared by Messrs. Lappe, Davidson, and Notess for the 27th Annual Meeting of the Institute of Aeronautical Sciences.** Subsequently, this paper was presented as a Task Report under NOas 58-517-d.

*Notess, C. B.: Analysis of Turbulence Data Measured in Flight at Altitudes up to 1600 Feet Above Three Different Types of Terrain. CAL Report No. TE-1215-F-1 April 10, 1959.

**Lappe, U. O., Davidson, B., and Notess, C. B.: Analysis of Atmospheric Turbulence Spectra Obtained from Concurrent Airplane and Tower Measurements. IAS Report No. 59-44, January 26-29, 1959.

TABLE OF CONTENTS

SYMBOLS

$\bar{U}$	mean wind speed, fps
V	true airspeed relative to mean air mass, fps
κ	wave-number
λ	wave-length, ft
ω	circular frequency, rad/sec.
Ω	reduced frequency, rad/ft.
σ	standard deviation of velocity fluctuations
$\Phi(\Omega)$	power spectral density, $(\text{ft/sec})^2$/rad/ft.
L	turbulence scale parameter, ft.
$F(\Omega)$	longitudinal (head-on) spectrum function
G	transverse (vertical and lateral) spectrum function
w	vertical component of turbulence velocity, fps
u	head-on component of turbulence velocity, fps
v	lateral component of turbulence velocity, fps
	unprimed values refer to airplane measurements; primed values to tower measurements

INTRODUCTION

The spectral density behavior of atmospheric turbulence in the height layer 10 to 300 ft. above ground have been studied by Panofsky and Vander Hoven (1956), Deland (1957), and others. For the most part these studies have been confined to the Brookhaven, Long Island area which represents relatively flat terrain, and the O'Neill, Nebraska area which represents very smooth terrain. Turbulence spectra at greater height levels have been measured by airplane techniques since 1949 by CAL, MIT, NASA, and others.

The meteorological turbulence spectra obtained from measurements at a fixed point on meteorologically instrumented towers represent spectra in the time or frequency domain. The spectra obtained from aircraft measurements represent spectra in the space domain. If the hypothesis of G. I. Taylor is valid for atmospheric turbulence, the meteorological time spectra can be converted into space spectra. Based on the results of the restricted meteorological studies, it appears that the following parameters are important in describing the behavior of atmospheric turbulence spectra.

1. Mean wind speed.
2. Atmospheric stability (lapse rate).
3. Altitude above mean terrain level.
4. Terrain characteristics.
5. Flight path direction relative to terrain features and mean wind direction.
6. General weather conditions (e.g. frontal conditions).

The objective of atmospheric turbulence research, insofar as aircraft application is concerned, is to relate turbulence spectral density behavior to the principal meteorological parameters and terrain classifications. In other words, to construct a minimum parameter turbulence model that might consist of the presently used spectral shape (scale of turbulence) and intensity (total variance) factors, parametrically related very probably to the mean wind speed, altitude, lapse rate, and terrain character.

The assumption presently used in aircraft design work is that the power spectra can be approximately represented by empirical expressions suggested by Dryden (1939) on the basis of wind tunnel turbulence measurements. These formulations contain two parameters that are assumed to characterize atmospheric spectra; namely, a characteristic length scale of turbulence and an intensity (standard deviation) parameter. Assuming these or similar formulations to be representative of the form of the power spectra it remains to determine how these parameters are related to wind speed, altitude, lapse rate, and terrain conditions.

The aim of the tri-tower experiments was to shed some light on this parametization problem through the utilization of concurrent tower measurements. The limited scope of the experiment precluded the possibility of actually providing the data necessary for a definitive model. However, some of the effects of the parameters that are believed to be more significant for turbulence at low altitude, were isolated.

TRI-TOWER EXPERIMENT

Objectives - The joint tower-airplane experiment was designed to provide concurrent turbulence data for one flight level (at or near tower height) and at the same time to supplement these data with airplane measurements above, and tower measurements below the tower height. Three tower locations were incorporated in this experiment in an attempt to determine the effects of different topographical conditions on the turbulence spectrum. The four principal objectives were:

(1) Obtain data to test the hypothesis of G. I. Taylor (1937) that turbulence encountered along a path in space aligned with the mean wind can be simply related to turbulence measured at a fixed point as a function of time.

(2) Obtain data for examining low level temporal and spatial homogeneity.

(3) Investigate the influence of terrain variations on the turbulence power spectral densities.

(4) Investigate the variation of turbulence characteristics with height above ground.

Aircraft Instrumentation - The aircraft measurements were made with a McDonnell FH-1 airplane instrumented with a differential pressure tube arrangement, the output of which provides the relative wind velocity along the flight path and the angular deviations (angle of attack and angle of side slip) of the relative wind vector in the vertical and horizontal planes. These measurements are with reference to a coördinate system fixed in the airplane and must be corrected for pitch, yaw, and linear translation of the airplane, i.e., the aircraft motion relative to the ground. The expressions used are summarized in Appendix I, and the technique is described in greater detail by Notess (1957), Lappe (1955) and Chilton (1954). After correction and reduction, the aircraft measurements provide time histories of the 3 orthogonal components of the turbulent velocity resolved on axes defined by the flight path through space. These components are denoted by u, w and v, where u is the longitudinal component, w is the vertical and v is the lateral component of the turbulent velocity. The turbulence spectra were computed for wave-lengths extending from about 100 to 6,600 feet.

Tower-Instrumentation - Turbulent fluctuations of the velocity vector were measured on meteorological towers by the use of a vane free to rotate in the azimuth and vertical planes, and an anemometer which measures the magnitude of the velocity vector. The mean wind is substracted from the data, and the turbulent fluctuation history is reconstructed by multiplication of the velocity vector by an appropriate function of the vane angular reading. The three orthogonal components of velocity are referred to the mean wind axis u' being the longitudinal component along the mean wind axis, v' the lateral and w' the vertical component of velocity.

The instruments used at Brookhaven consisted of a bivane, described by Mazerella (1952), operated in conjunction with a standard Bendix-Friez Aerovane. Because of a vane resonant frequency of about one cycle per second, all Brookhaven data are time averaged for five seconds prior to spectral computations. The instruments used at Peekskill consisted of a sensitive bivane of the type described by Cramer, Gill, and Record (1953), operated in conjunction with a light Beckman Whitley cup anemometer. The time constant of the entire system is somewhat less than 1 second for moderate wind speeds. Data points at Peekskill were generally read at one or two second intervals.

Meteorological tower observations were made at 75, 150 and 300 ft. above the ground at Brookhaven and at 300 ft. above the ground at Peekskill.

Topography and Flight Paths - The Brookhaven, Peekskill and Round Hill sites were chosen not only because meteorological towers were already in operation at these points but also because the topography surrounding each tower was distinctly different and offered an opportunity of studying the effect of terrain on turbulent intensities. The mean wind direction for most of the operations was 310°.

(a) The Brookhaven site is mapped in figure 1. The terrain surrounding the site is reasonably flat with wooded areas alternating with cleared fields. The headings of the flight paths are indicated by the dashed lines. Because of flight restrictions, it was not possible to fly along and normal to the mean wind direction. Again, because of flight restrictions, the lowest altitude flown was 400 ft. above terrain as contrasted with an upper limit of 300 ft. above terrain for the meteorological tower measurements.

Due to the comparatively large scale uniformly of the terrain surrounding Brookhaven, it was felt that the Brookhaven flights afforded the best opportunity for investigating the validity of Taylor's hypothesis for low altitude turbulence. Since tower measurements were available at 75, 150 and 300 ft. above the terrain and airplane measurements were made at 400, 800, and 1600 ft., the variation of the spectrum with height over reasonably uniform terrain could be studied in the range 75 to 1600 ft.

(b) The Peekskill site lies in the Hudson River Valley. The area is mapped in figure 2. A ridge about 900 ft. high lies about 2 miles to the NW of the tower. There is an abrupt drop to the Hudson River, which is close to a mile wide at this point. At the east bank of the river the terrain rises sharply to 100 ft. above river level. Near the final portion of the flight track the terrain rises to 500 ft. This stretch of land is mainly wooded but has some cleared areas. The overall terrain is certainly not homogeneous, and it was felt that this site offered an opportunity of studying the generation and decay of turbulence as the air passed over the mountain ridge and thence over the river and past the tower.

Flight tracks therefore were selected as shown by the dashed lines in figure 2. The flight runs at each tower location are summarized in Table 1. The 135-310° passes were made at 1,000 ft. above the river over terrain which varied sharply both in large scale roughness elements (the ridge) and small scale roughness elements (the river, wooded areas, etc.). The 035-230° tracks were flown at 400 and 1,000 ft. above the river. The 400 ft. tracks correspond to the altitude at which the tower measurements were made. These tracks are somewhat more homogeneous with respect to large scale terrain features than the 135-310° tracks, but large spatial gradients of turbulence might also be expected with this flight pattern.

(c) The Round Hill terrain is mapped in figure 3. The distinctive feature here is the land-water discontinuity. Because of the small tower at Round Hill, no attempt has been made to reduce the Round-Hill meteorological data. The flight patterns are shown in the figure and were designed (1) to bring out the effect of the land-sea boundary on the turbulence encountered during the specified flight tracks, and (2) to investigate the variation of the spectrum with height at 150, 300 and 600 ft. above terrain. Flight runs at the 150 ft. altitude were made only over the water.

Meteorological Conditions During Experiments - Flight and tower runs were made from 0913-1120, 24 April 1956 at Brookhaven and Peekskill. On 30 April 1956 runs were made at Brookhaven, Peekskill and Round Hill from 1010 to 1700. On 24 April both Brookhaven and Peekskill were in light to moderate west to west-northwest flow. Skies were generally overcast with a high broken deck at 0830 lowering to overcast estimated at 7,000 ft. by 1330. The following conditions were observed at the towers during airplane runs for 24 April.

Time of Airplane Runs		Mean Wind Speed at 300 ft. level	Temp. Gradient	Temp. Level, ft.
Brookhaven	0913-0935	21.6 fps.	1.3C/100m	410, 37.5
Peekskill	1100-1120	21.8 fps.	2.5C/100m	300, 7.5

On 30 April 1956 a rapidly moving cold front passed Brookhaven at about 0830 and flights were made according to the schedule listed below. A thin broken to overcast deck of high clouds persisted after the front passed. The following conditions were observed at the towers during the airplane runs for 30 April.

Time of Airplane Runs		Mean Wind Speed at 300 ft. level	Temp. Gradient	Temp. Level, ft.
Brookhaven	1010-1030	45.5 fps.	1.4C/100m	410, 37.5
Peekskill	1145-1220	30.3 fps.	1.9C/100m	300, 7.5
Round Hill	1400-1420	-----	----	
Round Hill	1640-1700	-----	----	

SPECTRUM REPRESENTATION BASED ON TAYLOR'S HYPOTHESIS

The spectral technique of Tukey (1956) was used to analyze the airplane and tower data. A summary of the computation design details used in the reduction of the airplane and tower turbulence data are presented in Appendix II.

In the spectral analysis, one-dimensional time histories, of u, v, and w along the flight path and at the tower were transformed to a wave-number (reciprocal wave-length) domain by utilizing Taylor's hypothesis (1938). This hypothesis states that in a homogeneous and isotropic field the space variation of turbulence can be determined by considering the turbulence at rest (or frozen) and moving the observer through the field with a relative velocity -U, where U is the velocity of the air stream transporting the turbulence. For the tower data U is the mean wind speed, $\bar{U}$. That is,

$$u'(t) = u'\left(\frac{x}{\bar{U}}\right)$$

The hypothesis is based on the argument that the standard deviation of the fluctuations are much smaller than the mean speed, i. e., $\overline{u'^2}/\bar{U}^2 \ll 1$

For the airplane turbulence data, Taylor's hypothesis assumes that

$$u(t) = u\left(\frac{x}{V}\right)$$

where V is the velocity of airplane relative to the mean air mass. Since V is much larger than $\bar{U}$, it follows that

$$\frac{\overline{u^2}}{V^2} \ll \frac{\overline{u'^2}}{\bar{U}^2}$$

whence application of Taylor's hypothesis to the airplane data is not nearly as critical a procedure as application of the hypothesis to the tower data. In fact, the airplane very nearly measures the true space spectra.

The tower and airplane wave number spectra for u, v, and w were obtained by computing spectral densities (per rad/sec) in the frequency domain and converting by the appropriate velocity to a wave number domain.

$$\frac{\omega}{\bar{U}} = \frac{2}{\lambda} \quad \text{or} \quad 2\pi\kappa = \Omega \text{ (rad/ft.)}$$

where ω is the circular frequency, λ the wave length, and κ the wave number. Because of the change in the density argument from rad/sec to rad/ft., the spectrum function $\Phi(\omega)$ must be multiplied by $\bar{U}$. Thus,

$$\bar{U}\,\Phi(\omega) = \Phi(\Omega), \text{ (ft./sec)}^2\text{/rad/ft.}$$

DISCUSSION OF RESULTS

Temporal and Spatial Homogeneity - Temporal homogeneity implies time invariance (stationarity) of the spectrum. Spatial homogeneity implies that the statistical parameters, such as mean square velocity (total variance) and the spectrum, are invariant under a translation of the coordinate axes. Because of the effect of the ground on the vertical distribution of turbulence, atmospheric turbulence near the ground is likely to be homogeneous for horizontal translations only, provided of course that the underlying terrain is also reasonably homogeneous. If atmospheric turbulence is homogeneous for horizontal translations and stationary for periods of at least several minutes then it may be expected that the spectrum of the velocity components determined from flight data should be identical (save for sampling variations), for repeat runs with identical headings. A change or orientation of the flight track through space is equivalent to a rotation of the axes on which the components of the velocity vector are resolved. If the turbulence were isotropic, then, save for sampling variations, the spectrum of the velocity components, u, v, and w, resolved on the coordinate system defined by the flight path should not change as the flight heading is changed.

It was not expected that low altitude turbulence would be homogeneous with respect to vertical translations or isotropic with respect to rotations about a horizontal axes. The question of how nearly the spectrum exhibits the properties of homogeneity with respect to horizontal translations, and of isotrophy with respect to rotation about a vertical axis, is important. Therefore flight patterns were arranged to have repeat runs made at the same headings, and also along intersecting flight headings as shown in figures 1, 2, and 3.

A comparison of the w spectrum obtained for repeat runs and for intersecting headings at the 800 ft. level at Brookhaven where the terrain is reasonably homogeneous is shown in figure 4. The data indicates that differences in the spectra occur chiefly in the low wave number region ($\Omega < .006$ rad/ft.). For the particular flights shown, the differences in repeat runs are less than the differences due to the heading changes. Analysis of other flights indicate, however, that this is not the case generally and that on the average the differences in repeat runs are about the same as the differences caused by heading changes. It is believed that the differences in the low wave number region of the spectrum can be attributed to sampling variations. To test this hypothesis the F distribution was used. This distribution function is obtained from a ratio of sample variances known to possess independent chi-square distribution. In the present case, the sample variances are the independent spectral density estimates obtained for repeat runs and for different headings. Applying the F distribution to the low wave number variances shown in figure 4, variance ratios between the 60° and 270° headings (Φ_{60}/Φ_{270}) of approximately 2, 1.9, and 1.7 are obtained at Ω values of 10^{-3}, 1.5×10^{-3}, 1.5×10^{-3}, and 1.7×10^{-3} respectively. From a table of F distributions based on 95% confidence, a value of 2.18 is obtained for 30 degrees of freedom. Since this theoretical ratio is greater (although not much greater) than the observed ratios of sample variances, there is reason to assume that the observed differences are due to sampling variations. For the u and v components similar low wave number variances were noted.

At Peekskill, the roughness of the terrain and the limited number of flights obtained made it more difficult to examine the variability of the spectra for repeat runs and for changes in flight path direction. Nevertheless, repeat runs at 400 ft. (see figure 2 for flight paths) indicated spectral variability results comparable to those noted for Brookhaven. Direction changes at 400 ft. could not be examined because of the high ridge.

For the 1,000 ft. level at Peekskill, repeat runs at 135° and 310° on April 30th (figure 5) indicates a rather pronounced change in spectral shape for the 135° downwind track. The nature of this spectral shape change is such as to produce about a 75 per cent increase in spectral variance for the 135° downwind flight, when compared to the 310° repeat run (see Table I).

On April 24th little change in spectral shape occurred at either altitude, but the 135° downwind flight at 1,000 ft. again indicated a spectral variance increase over the upwind flight; this time about 40 per cent. Since there was no apparent reason to suspect instrumentation malfunction for these two downwind flights, it was surmised that this seemingly anomalous behavior was indicative of what may occasionally occur over confused terrain.

For the 1,000 ft. level it was also possible to compare heading changes (230° vs. 310°) on the 30th of April. This comparison is shown in figure 6.

The differences observed for these two flight directions, both in shape and spectral variance (25.2 vs. 14.6), is again attributed to the extreme terrain variations at Peekskill.

At Round Hill, where the terrain is reasonably homogeneous, it was found that intersecting flight paths, at the same altitude, showed little spectral variations. Repeat flights were not made at Round Hill.

It follows then that where the terrain is reasonably homogeneous, the atmosphere exhibits some of the properties of homogeneity (both spatial and temporal) with respect to horizontal translations, and some of the properties of isotropy, at high wave numbers, with respect to rotations about the vertical axis. These are very important properties from the sampling point of view, and, if confirmed by additional observation, indicate that the number of samples required to determine spectral behavior during unstable atmospheric conditions over reasonably homogeneous terrain, is much less than would be required if atmospheric turbulence did not possess, to so large an extent, the properties of homogeneity and isotropy.

Considering the tower data, it seems evident that stationarity is much less likely to be indicated by the towers than by the airplane. This is because the wave lengths being sampled by the airplane require tower record lengths ranging up to one hour periods; successive records under these conditions usually reflect changing large scale meteorological conditions. However, successive tower spectra, when normalized by the total variance, show little change in form, indicating some stationarity of the form of spectrum, if not of the total variance.

Local Isotropy - It is evident that general isotropy, which requires the statistical properties to be independent of the rotation and reflection of coordinate axes for the entire spectral range, does not exist in the atmosphere. However, the notion of local isotropy, first put forth by Kolmogoroff (1941), postulates the existence of a restricted inertial subrange of wave numbers in which the velocity components are locally isotropic. The inertial subrange should exist in flow characterized by a high Reynold's number, provided there is a wide separation on the wave number axis between the eddies containing most of the energy and the eddies responsible for most of the frictional dissipation. In the locally isotropic range the three dimensional energy spectra should exhibit a -5/3 slope; the one-dimensional u, v, and w spectra should also exhibit a -5/3 slope.

In the atmosphere the criteria for the existence of the inertial subrange are generally fulfilled, provided the measurements are not too close to the ground. It was therefore expected that both airplane and tower spectra would exhibit some degree of local isotropy. Furthermore, if Taylor's hypothesis of the equivalence of space and time spectra for isotropic turbulence is valid in the atmosphere, both airplane and tower spectra should agree in the isotropic range. Examining the airplane (figure 7) separately for local isotropy indicates, generally, that the airplane spectra for the three velocity components are about equal for $\Omega > .01$ (wave lengths less than 600 to 1,000 feet) with slopes approaching -2 on a log-log plot (see next section). Examples of the tower spectrum components for Brookhaven and Peekskill are shown in figures 8 and 9.

Power Law Behavior at High Frequencies - The present airplane and tower data permit a fairly direct comparison of not only the tower versus airplane data, but also of the two sets of instruments employed at the Brookhaven and Peekskill towers. Examining the high frequency behavior of the spectral results between tower and airplane at Brookhaven indicates that the tower results are best approximated by a -5/3 power law, while the airplane spectra are better approximated by a -2 law. The Peekskill tower spectra, however, appear to be in better agreement with the airplane spectra in this high frequency range.

In an effort to shed additional light on the power law behavior, an attempt was made to utilize the characteristics of the autocorrelation function at small lag values.

It is generally recognized that the autocorrelation function is much less desirable than the spectrum function in characterizing a random process, principally because the estimates are not statistically independent and the sampling fluctuations effectively mask the large wave length behavior. Conversely, however, the covariance behavior at small lags will determine the spectrum behavior at high frequencies. It is this property that is believed useful in providing a more accurate indication of whether the -5/3 or -2 power law better describes atmospheric turbulence spectrum behavior in the inertial subrange.

We note that the autocorrelation function for the empirical spectrum

$$G(\Omega) = \frac{\sigma^2 L}{\pi} \; \frac{1 + 3\,\Omega^2 L^2}{(1 + \Omega^2 L)^2}$$

is

$$g(r) = (1 - \frac{r}{2L})\, e^{-r/L}$$

where r is the correlation distance, in feet, and L the scale of turbulence.

Similarly, for the Kolmogoroff spectrum

$$G(\Omega) = C\Omega^{-5/3}$$

defined for $\Omega > \Omega_d$, where Ω_d is the beginning of the intertial subrange, the autocorrelation is of the form

$$g(r) = 1 - B\, r^{2/3}$$

where $r < r_d$ and $r_d \approx \frac{\pi}{\Omega_d}$

Examining the behavior of $G(\Omega)$ for $\Omega > \Omega_d$ is equivalent to examining $g(r)$ for $r < r_d$. Thus, if the -5/3 law better describes atmospheric turbulence, the computed normalized autocovariance values should show better agreement with $1 - Br^{2/3}$ than with $(1 - \frac{r}{2L})e^{-r/L}$ for $r < r_d$.

In this analysis, comparisons were made of the normalized autocovariance coefficients computed for the Brookhaven and Peekskill towers and for the airplane. A number of values for the constants B and C were selected to provide a family of autocorrelation functions to compare with computed coefficients. Both the tower and airplane were converted from time to space iags according to $\Delta r = \bar{U}\Delta t$, where $\bar{U}$ is either the mean wind speed or true airplane speed.

For the Brookhaven data, it was found that only one or two computed autocovariance coefficients fell within the range $r < r_d$, where the $1 - Br^{2/3}$ function might be considered valid. Thus, with only one or two coefficients available, either correlation function could be made to fit. This result was generally expected because of the poor high frequency resolution of the more rugged bivane and aerovane instruments used at Brookhaven, but it is interesting to note that it is more evident in the correlation form than in the spectrum form.

For the Peekskill tower, figures 10 and 11 compare four sets of data obtained during the Tri-Tower operation. It is apparent that the $(1 - \frac{r}{2L})e^{-r/L}$ function is a much better fit for the computed coefficients than the $1 - Br^{2/3}$ function. A similar comparison is presented by figures 12 and 13 for the airplane data.

On the basis of these data it is concluded that (1) the spectrum behavior during unstable conditions in the atmosphere, for the inertial subrange of frequencies, is closer to a -2 power law than a -5/3 law, and (2) fast responding meteorological instruments compare favorably with properly desgined aircraft turbulence instrumentation.

Taylor's Hypothesis - One of the principal objectives of the Tri-Tower experiment was to determine the validity of Taylor's hypothesis. If the hypothesis is valid, tower meteorological measurements can be used in describing the turbulence encountered by aircraft. While the hypothesis seems to have been verified for wind tunnel turbulence levels, some doubt has been expressed as to the validity of the hypothesis when applied to the atmosphere.

Lin (1953) has discussed the validity of applying Taylor's hypothesis under shear flow conditions. His conclusions are that the hypothesis is valid in shear flow only for wave numbers such that

$$2\pi\kappa \text{ (or } \Omega) >> \frac{2\pi}{\bar{U}} \frac{d\bar{U}}{dZ}$$

where κ is the wave number and $d\bar{U}/dz$ the wind shear at flight altitude. Evaluating $d\bar{U}/dz$ from the log-law, which has been found valid in the atmosphere for neutral conditions near the ground, it follows that

$$\Omega >> \frac{2\pi}{h \ln(\frac{h}{z_o})}$$

where h is the height of observation, and z_o a roughness parameter. Values of z_o for both the Brookhaven and Peekskill installations are estimated to be on the order of 3. 3 ft., whence

$$\Omega >> 0033 \text{ rad/ft.}$$

for flights at 400 ft.

According to this criteria, Taylor's hypothesis for the experiments performed at Brookhaven and Peekskill should be valid for wave lengths much less than 2, 000 ft. Strictly speaking, the airplane and tower spectra should be compared only for the case where the flight path is along the mean wind axis. At Brookhaven the flight paths were close to the mean wind direction, but at Peekskill the flight at tower level was restricted by the terrain to headings which were approximately normal to the mean wind axis. As indicated previously for Brookhaven, however, spectral changes with flight path direction appear no greater than the variability for identical flight paths.

Figures 14 and 15 compare the space spectra as reconstructed from concurrent airplane and tower data at Brookhaven and Peekskill. In general, the tower and airplane spectra are in approximate agreement for the higher wave number region of the spectrum. At lower wave number $\Omega < .01$ radians/ft. (wave lengths > 600-1, 000 ft.), the tower data indicate less turbulent energy than the airplane spectral measurements. It is difficult to determine the exact reason for this on the basis of the present experiment, although several possible explanations can be advanced. These include (1) the difference in altitude between the 400 ft. airplane data and (2) the possibility that the airplane is measuring energy associated with terrain roughness that is not present in the vicinity of the tower (this seems definitely to be the case at Peekskill), and that this energy has an opportunity to decay in the time it takes the mean wind to transport the long wave length energy to the tower.

The extent of decay depends on the distance upstream from the tower that the long wave energy was introduced and the mean wind speed. Thus, a greater discrepancy would be expected between airplane and tower spectra at low wind speed conditions. A comparison of the results for the 24th and 30th (figure 14a) appears to support this hypothesis.

A closer examination of the 100 ft. height discrepancy between tower and airplane data indicates that the spectral differences for the 30th at Brookhaven (figure 16) appear roughly in agreement with a linear extrapolation of the tower spectra (75, 150, and 300 ft. data) to the 400 ft. airplane level. The differences between the tower and airplane spectra at Brookhaven on the 24th are much greater and cannot be explained on this basis.

Spectral Variations with Altitude - (a) Brookhaven - In figures 16 and 17 the respective w spectra for the 24th and 30th of April are shown for both the airplane and the tower. These figures show the spectral changes with altitude from 75 to 1,600 ft. (averaged values have been used where two or more spectra are available for the same height). In general, the behavior with height appears to involve an increase in low frequency energy components as the distance above the ground is increased, accompanied by a small decrease in high frequency energy. At low levels, this behavior results in an overall increase in spectral variance with increasing height. As the height is increased further, however, a condition is reached wherein the spectral variance will decrease with additional height increases - even though the spectral shape may not change. This change with height may be expected to depend on existing meteorological conditions. Figures 16 and 18 indicates that the April 24th spectra is described by this general behavior, and the transition altitude would appear to occur somewhere between 800 and 1, 000 feet.

Analysis of the spectra for April 30th is more involved. Of the six runs made on April 30th, five were reduced to spectral form. Figure 18 shows the spectral variances obtained from these five runs. The variance for run 5 (at 1, 600 ft.) is nearly twice that for run 6 and appears unusually high. This large spectral variance difference is probably indicative of the extreme variability possible in a post cold frontal condition. These runs were made about 1-1/2 hours after the cold front had passed. For this reason, and because the one 400 ft. spectrum obtained indicates considerable more spectral energy than the 300 ft. tower data, the height-spectral energy variation cannot be easily compared with the 24th. However, if the spectra for the one 400 ft. and two 800 ft. runs for the 30th are plotted with run 6 at 1, 600 ft., a spectral shape variation with height roughly similar to April 24 is obtained, viz, a strong shape dependency with height up to 800 ft., and very little change thereafter.

To summarize the effect of altitude on these vertical spectra, both airplane and tower estimates were divided by values obtained by integrating the estimates from $\Omega = .01$ to $\Omega = .06$. This normalization of the higher wave number spectral values has the effect of reducing spectral differences between these wave numbers and emphasizing the low wave number variation with altitude. The average (and faired) estimates thus modified for the 24th and 30th, are summarized in figures 16 and 17. The turbulence scale increase with altitude is clearly shown by these modified estimates.

(b) Peekskill - Figure 19 shows a comparison of the 24th and 30th spectra obtained at 400 and 1,000 ft. The principal difference noted is an increase in turbulence scale as the altitude is increased from 400 to 1,000 ft. That is, the 400 ft. spectra have a greater low frequency curvature than the 1,000 ft. spectra. Modified Peekskill spectra, obtained by normalizing the high frequency values from $\Omega = .01$ to $\Omega = .06$, as at Brookhaven, are shown in figure 20.

(c) Round Hill - Figure 21 shows the w spectra for over-water flights at 150, 300, and 600 ft., and land to water flights at 300 to 600 ft. Differences in spectral energy between the land-to-water and over-water flights are seen to be about 3 to 1. Height variations appear roughly as follows:

(1) Land to water runs - Principally an increase in turbulence scale in going from 300 to 600 ft.

(2) Over water runs - A gradual increase in low frequency spectral energy with increasing height. This increase amounts to about 30 per cent as the runs are increased from 150 to 600 feet.

Effect of Terrain on Spectra - To obtain some indication of the degree of correlation of the turbulence with terrain features, variances were computed for one mile segments at the three tower locations.

At Peekskill one mile variances were computed for the 135 degrees and 310° flight paths at the 1,000 ft. level; these were averaged for the 24th and 30th. The results are shown in figure 22 and show a rather sharp decrease in the turbulence level over the river, and near the tower. Downwind from the tower the turbulence increases again as the terrain becomes more irregular.

At Round Hill the one mile variances were computed for flights at 300 ft. and flight headings of 180° and 155°. These are shown in figure 23 as a function of distance from the coast line, in the mean wind direction. An abrupt decrease in turbulent energy of about 4 to 1 occurs at 3 to 4 miles from the coast line. The effect of flying by the Naushon Island during the 155° flight is also shown.

At Brookhaven the terrain is relatively flat; computed variances for one mile segment at this location for the 1,600 ft. - 270° flight track on April 24th indicated that considerable energy variation may take place even in the absence of rough terrain features. Averages over longer segments appear to average out these effects, as do repeated runs. Over highly irregular terrain, such as the Peekskill site, the one mile variances tend to follow the profile features of the terrain. This indicates that the energy maxima are very closely associated with the large scale terrain features and are not transported large distances downstream by the mean flow. For the land-water flight track at Round Hill, however, the turbulence is carried out to sea by the mean flow for some three or four miles before decreasing abruptly. The seaward persistence of the energy level may be caused by the absences of ground friction effects on the turbulent motion. It is evident from the Peekskill-Brookhaven flights during similar meteorological conditions (figure 24), that rough terrain features will contribute to an increase in total energy. Figure 24 shows that the energy increase occurs primarily at the low frequencies and must undoubtedly be the result of the larger terrain features found at the Peekskill site. Similar results were found in flights over the Boston Hill near Buffalo, N. Y. by Notess (1957).

Wind Speed and Stability Effect on the Spectra - The principal effect of increasing wind speed is to increase the total energy. Total variances computed from the tower and airplane data taken at Brookhaven and Peekskill on the 24th and 30th are shown in figure 25. The airplane and tower measurements show a roughly similar variation of σ_w^2 and U, namely $\sigma_w^2 \propto \overline{U}^{1.6}$. The effect of atmospheric stability changes cannot be defined with the Tri-Tower data, since only two unstable conditions were tested. However, from measurements made at the Peekskill tower on 18 September 1956 at 1400 when the atmosphere was unstable, and at 1800 when neutral conditions prevailed, it is possible to indicate the effect of convective conditions on the spectral behavior. In figure 26 are shown the normalized w spectra for these two stability conditions. It is noted that there is relatively less low wave number energy for the adiabatic conditions than for the unstable case.

Spectrum Representation by the F (Ω) and G (Ω) Functions - The present data is not sufficient to permit parametizing spectral behavior with changing conditions. We can, however, consider how the vertical velocity spectra vary in shape and intensity under the conditions tested. In figure 27, the spectra representing the over-land minimum and maximum spectra obtained during the tests are shown. These were obtained at Brookhaven April 24 (1600') and Peekskill, April 30 (400'), respectively. Within this spectral envelope are included variations with terrain, wind speed (20 to 40 fps), altitude (400 to 1, 600 ft.), and minor atmospheric stability changes. It is seen that these spectra cover a spectral intensity range of about 3 to 1 over most wave numbers. Since the two spectra in figure 27 are more or less typical, the emprical G(Ω) function was used in an attempt to fit the vertical spectra. With a scale length of 600 feet for the 1, 600 ft. Brookhaven spectra, the agreement shown in figure 27 was obtained.

For the 400 ft. Peekskill spectra, however, it was found that the spectral shape changed too rapidly at the low wave number values to permit an adequate representation by the $G(\Omega)$ function. Therefore, the $F(\Omega)$ function, which varies less rapidly at low wave number than $G(\Omega)$ was tried. The agreement obtained with the $F(\Omega)$ function, with a scale length of 400 ft., is also shown in figure 27.

In general, it appears that neither the $F(\Omega)$ or $G(\Omega)$ functions provides a satisfactory representation for the spectra obtained. The spectral shapes appear to have less curvature (at low frequencies) then allowed by either $F(\Omega)$ or $G(\Omega)$. Attempts to fit the spectra by a slightly modified function of the form $(1+\Omega L)^{-2}$ appeared to offer slight improvement. Henry (1959) has utilized the hyperbolictangent in representing some of the Brookhaven and O'Neill tower data. A preliminary examination of the hyperbolictangent with respect to the present airplane-tower data indicates that it may prove to be a better empirical form to use for spectral representation.

CONCLUDING REMARKS

The Tri-Tower experiment does not cover a sufficient range of topographical or meteorological conditions, nor provide a sufficient large statistical sample, to permit an extrapolation of the results to establish general laws of spectral behavior with changing conditions. It is believed, however, that the results of these experiments will serve as a guide in the design of larger atmospheric turbulence programs, such as the current B-66 turbulence project being conducted by Douglas Aircraft Company.

Some of the more important characteristics of the power spectral density behavior, for the conditions under which the measurements were made, are:

(1) Turbulence over reasonably homogeneous terrain (such as Brookhaven) appears to be homogeneous in the horizontal plane and to exhibit isotropy with respect to rotations about a vertical axis.

(2) Taylor's hypothesis, that space spectra and time spectra at a point are related in wave number space, is verified in the high wave number range of the spectrum.

(3) Over homogeneous terrain or over water the vertical energies in the low wave number range appear to increase with altitude, while the high wave number spectral densities tend to decrease slightly with altitude. The variation of the mean square values with altitude could not be established.

(4) As the terrain becomes rougher, turbulent energy increases. Conditions typified by the tower locations indicated a 20 % increase at Peekskill and 50 % decrease at Round Hill (land-water condition), relative to Brookhaven. The differences are primarily

the result of changes in the low wave number content of the turbulence.

(5) Turbulent energy carried seaward by an off shore wind may persist for several miles, before abruptly diminishing to a fraction of the over-land value. A four to one ratio was noted at Round Hill.

(6) For terrain characterized by large horizontal variations of roughness elements, the mean square fluctuations, computed over one mile segments, correlate well with the topographical profile.

(7) On the basis of both airplane and tower spectra the vertical turbulence energy (σ_w^2) increases with mean wind speed raised to the 1.6 power, approximately.

(8) Peekskill tower data indicate that under essentially the same synoptic conditions, the mean wind speed decreased by a factor of 30% while the turbulent energy decreased by a factor of nearly 85% as the atmosphere changed from an unstable to a neutral equilibrium condition. The spectral shape is not altered at the low wave numbers where comparatively less energy is measured.

REFERENCES

1. Panofsky, H. A. and Vander Hoven, I.: Structure of Small Scale and Middle Scale Turbulence at Brookhaven. AFCRC-TN-56-254, March 1, 1956.
2. Deland, R. J., and Panofsky, H. A.: Structure of Turbulence at O'Neill, Nebraska and its Relation to the Structure at Brookhaven. AFCRC-TN-57-262, January 1, 1957.
3. Taylor, G. I.: The Spectrum of Turbulence. Proc. Roy. Soc. 164 (A), 1938.
4. Notess, C. B.: A Study of the Nature of Atmospheric Turbulence Based Upon Flight Measurement of the Gust Velocity Components. WADC TR 57-259, April 23, 1957.
5. Lappe, U. O.: A Direct Method of Utilizing Flight Data to Determine Space and Spectrum Gust Velocity Distributions and Airplane Gust Performance Function. C. A. L. GM-776-T-45 August, 1955.
6. Chilton, R. G.: Some Measurements of Atmospheric Turbulence Obtained from Flow-Direction Vanes Mounted on an Airplane. NACA TN 3313, November 1954.
7. Mazzerella, D. A.: Wind Tunnel Tests on Seven Aerovanes. BNL, 1953.
8. Cramer, H. E., Gill, G. C., and Record, F. A.: Heated-Thermocouple Anemometers and Light Bivanes - Massachusetts Institute of Technology. Vol. 1. Exploring the Atmosphere's First Mile.
9. Kolmogoroff, A. N.: The Local Structure of Turbulence in Incompressible Viscous Fluid for Very Large Reynolds Numbers. C. R. Acad. Sci. U.R. S.S. 30, 301, 1941.

10. Lin, C. C.: On Taylor's Hypothesis and the Acceleration Terms in the Nairer-Stokes. Q. A. M. Vol. X, January, 1958.

11. Press, H., and Tukey, J. W.: Power Spectral Methods of Analysis and their Application to Problems in Airplanes Dynamics. AGARD Flight Test Manual Vol. IV, Part IVC, June, 1956.

12. Dryden, H. L.: Turbulence Investigation at the National Bureau of Standards. Proc. 5th Int. Congr. Appl. Mech. p. 366, 1938.

APPENDIX I

Flight Measurement of Gust Velocity

The gust velocity vector can be specified by its components resolved along three orthogonal axes defined as follows:

Axis	Gust Velocity Component
X	u_{ag}, horizontal component in average direction of flight path
Y	v_{ag}, horizontal velocity component in a direction perpendicular to X and Z axes
Z	w_{ag}, vertical velocity component

The basic instrument used for flight measurement of the gust velocities was a differential pressure type of flow direction probe. The probe had a one inch diameter hemispherical head with pressure ports so located as to provide total head and static pressures and differential pressures proportional to the angles of attack and sideslip. As an example, the angle of attack is related to differential pressure and measured impact pressure according to the following equations:

$$\Delta \alpha = k \frac{\Delta p}{1/2 \, \rho \, V^2} \tag{1}$$

where

$\Delta \alpha$	incremental angle of attack
Δp	differential pressure
ρ	ambient air density
V	airplane speed relative to air

The flight data runs were made at essentially constant airspeed and altitude so that ρ and V in equation (1) were replaced by constant average values during each run.

The gust velocity components were determined from the equation

$$V_{ag} = V_{pg} - V_{pa} \qquad (2)$$

where:

V_{ag}	represents velocity of air relative to ground
V_{pg}	represents velocity of probe relative to ground
V_{pa}	represents velocity of probe relative to air, henceforth denoted as V.

The velocity of the probe relative to the ground was obtained by measurement of angular velocities with rate gyros and measurement of linear accelerations with linear accelerometers.

The equations necessary to compute the time histories of the gust velocity components are described below.

1. u_{ag} - head-on gust velocity, positive in a forward direction.
   The frequency range of interest in these programs was of the order .067 to 5.0 cycles per second. The airplane speed for the data runs was kept constant to within $\pm 5\%$ of 300 mph indicated airspeed. This velocity transforms the frequency range to an equivalent wave length range of 88 to 6,600 feet. In this frequency range the change in forward speed of the FH-1 airplane with respect to the ground is very small. For this reason it is possible to approximate the head-on gust velocity by measured changes in airspeed. At 300 mph compressibility effects can be neglected and the change in airspeed can be obtained by measurement of incremental dynamic pressure (total head pressure minus static pressure).

$$u_{ag} = - \frac{\Delta H_c - \Delta p_s}{\rho \bar{V}} \tag{3}$$

ΔH_c	incremental total head pressure
Δp_s	incremental static head pressure
ρ	ambient air density
$\bar{V}$	average airspeed during data run, in this case about 440 ft/sec.

The altitude above mean sea level was maintained constant to within ± 100 feet during data runs. To account for such altitude variations affecting the value of ρ a correction term was derived using the pressure - density relation for the standard atmosphere:

$$\rho/\rho_{S.L.} = (p_s/p_{s_{S.L.}})^{.81} \tag{4}$$

The final equation used for calculating head-on gust velocity was therefore

$$u_{ag} = - \frac{\Delta H_c - \Delta p_s}{\bar{\rho} \bar{V}} + .405 \bar{V} \left(\frac{\bar{\rho}}{\rho_{S.L.}}\right)^{-1.23} \left(\frac{\Delta p_s}{p_{s_{S.L.}}}\right) \tag{5}$$

2. v_{ag} - side gust velocity, positive in direction of right wind. The horizontal velocity of air relative to the probe in a direction perpendicular to the average flight direction can be approximated by the equation

$$v_{pa} = \bar{V} (\Delta \beta + \Delta \Psi) = \bar{V} \Delta \beta + \bar{V} \int_o^t \dot{\Psi} \, dt \tag{6}$$

where:

$\Delta\beta$ is incremental angle of sideslip measured at the probe as a constant times differential pressure.

$\Delta\Psi$ is incremental yaw angle about average flight path heading measured by yaw rate gyro as $\dot{\Psi}$, or angular yaw rate.

The similar component of probe velocity relative to the ground can be approximated by the equation

$$v_{pg} = \int_0^t (n_y + g\,\phi)\,dt + 21.6\,\dot{\Psi} - 2\,\dot{\phi} \tag{7}$$

where:

n_y is reading of lateral accelerometer, 2 feet above the cg.

g is acceleration of gravity ~ 32.2 ft/sec^2.

ϕ is angle of bank measured by altitude gyro.

21.6-is distance from cg. to differential pressure probe.

2 is height of lateral accelerometer above cg., in feet.

$\dot{\phi}$ is angular roll rate measured by rate gyro.

From equations (2), (6), and (7) v_{ag} can be obtained. The actual equation used in the calculations assumed that V could be replaced by an average value, $\bar{V}$.

$$v_{ag} = -\bar{V}\,\Delta\beta - \bar{V}\int_0^t \dot{\Psi}\,dt + 21.6\,\dot{\Psi} - 2\,\dot{\phi} + \int_0^t (n_y + g\,\phi)\,dt \tag{8}$$

3. w_{ag} - vertical gust velocity, positive in downward direction. The vertical gust velocity can be defined in a fashion similar to that for v_{ag}. The result is

$$w_{ag} = -\bar{V}\,\Delta\beta\alpha - 21.6\,\dot{\theta} + \int_0^t (n_z + \bar{V}\,\dot{\theta})\,dt \tag{9}$$

APPENDIX II

Spectrum Equations and Design of Computations

The use of generalized harmonic analysis techniques in the analysis of atmospheric turbulence as a random process has been reported by Press and Tukey (1956). Therefore this technique will not be described herein, other than to define the terms used to describe the statistical information pertaining to the experiment.

The autocovariance and power spectrum transform expressions used in the analysis of the tower and airplane turbulent fluctuations were defined as:

$$R_p = \frac{1}{n-p} \sum_{q=1}^{n-p} x_q x_{q+p} \qquad (p = 0, 1, 2 \ldots . m)$$

x_q gust velocity in ft/sec. at time q

n number of points in the sample

m number of time lags used to define

R_p units: (ft/sec.)^2

and

$$L_h = \bar{V} \frac{2\Delta t}{\pi} \sum_{p=0}^{m} \epsilon_p \cos \frac{hp\pi}{m} R_p$$

where

$$\epsilon_p = \begin{cases} 1, \text{ if } 0 < p < m \\ 1/2, \text{ if } p = 0 \text{ or } m \end{cases} \quad ; h = 0, 1 \ldots . m$$

The power spectral density L_h has the units $\frac{(\text{ft/sec.})^2}{\text{rad/foot}}$ and $\bar{V}$ is the average true airspeed, fps.

Before computing the Rp's, the time histories were subjected to a high pass numerical filtering process (prewhitening) in order to eliminate very low-frequency energy that might otherwise be diffracted by the L_h computations to higher frequencies as false energy. Following the Rp computation, the autocovariances were multiplied by the function.

$$\left[1 - \left(\frac{p}{m}\right)^2\right] \left(.58 + .42 \cos \pi \frac{p}{m}\right)$$

This multiplication has the effect of modifying the equivalent band pass filter (spectral window) imposed by the L_h estimate of the power density for each frequency band $\left(\frac{\text{independent power point}}{2}\right)$.

The following table gives experimental design information for the airplane and tower measurements.

		Towers	
	Airplane	Brookhaven	Peekskill
Record length (approx. average)	120 seconds	60 mins.	20-40 mins.
Sampling interval	0.1 seconds	5 sec. (av.)	1-2 sec.
Number of correlation lags	75	60	60
Folding frequency	5.0 cps.	.1 cps.	.5-.25 cps.
Independent power point spacing	.133 cps.	.003 cps.	.016-.008 cps.
Degrees of freedom per independent power point	32	24	40

TABLE I

SUMMARY OF FLIGHTS AND TURBULENT VELOCITY VARIANCES

SITE/DATE	RUN #	ALT. (FT)	HDG (DEG)	σ_w^2	σ_u^2	σ_v^2
Brookhaven Apr. 24	2	400	280	8.4	9.2	23.2
	3	800	060	11.7	8.6	15.6
	4	400	060	11.3	10.6	18.5
	5	400	270	13.0	15.0	16.6
	6	800	060	13.6	7.3	16.2
	7	800	270	10.1	8.9	14.6
	8	1600	060	8.7	5.8	10.5
	9	1600	270	9.2	6.0	14.0
	Tower	300	---	7.7	13.2	11.9
Peekskill Apr. 24	1	1000	135	9.5	10.8	11.7
	2	1000	310	14.0	---	---
	5	400	230	16.8	14.4	15.6
	6	400	040	13.2	14.8	18.5
	Tower	300	---	6.8	9.0	8.5
Brookhaven Apr. 30	2	400	275	23.6	41.4	38.2
	3	800	055	18.4	21.2	38.7
	4	800	280	21.2	15.7	27.4
	5	1600	050	30.4	16.6	21.2
	6	1600	275	17.6	17.6	20.1
	Tower	300	---	17.3	22.4	20.8
Peekskill Apr. 30	1	1000	135	25.3	30.8	36.7
	2	1000	310	14.6	15.5	24.9
	3	1000	230	25.2	18.3	32.3
	4	1000	040	26.7	22.6	---
	5	400	240	24.8	23.9	49.7
	6	400	030	29.6	22.1	52.3
	7	400	230	28.7	---	36.0
	Tower	300	---	17.6	16.6	33.3
Round Hill Apr. 30 1st Flight	1	600	050	13.5	17.5	33.8
	2	600	170	11.1	17.5	12.3
	3	300	060	11.5	18.0	18.0
	4	300	180	12.6		
Round Hill Apr. 30 2nd Flight	1	300	050	3.3	6.5	6.1
	2	300	150	3.4	3.8	4.8
	4	600	160	3.8	6.3	7.0
	5	150	270	2.8	8.1	8.0

Note: σ^2 values listed for the towers were obtained from tower spectra which approximately correspond with the tower flights.

TABLE II (APRIL 24)

MEAN SQUARE VERTICAL VELOCITY VARIATIONS

(One Mile Intervals)

FLIGHT DESCRIPTION	MILES	$(w')^2$	2 MI. AVG.	4 MI. AVG.
Brookhaven	1	6.20		
1600'-270°	2	1.14	3.67	
	3	3.23		7.85
	4	20.80	12.02	
	5	6.78		10.20
	6	9.97	8.38	
	7	6.56		8.25
	8	9.66	8.11	
Peekskill	1	7.53		
1000'-310°	2	20.88	14.21	
	3	10.19		11.93
	4	9.08	9.64	
	5	21.69		12.71
	6	9.87	15.78	
	7	7.70		11.97
	8	8.61	8.16	
	9	11.62		10.37
	10	13.53	12.58	
Peekskill	1	15.59		
1000'-135°	2	7.31	11.45	
	3	9.90		9.07
	4	3.47	6.69	
	5	4.25		7.39
	6	11.93	8.09	
	7	6.18		

TABLE II (APRIL 30)

MEAN SQUARE VERTICAL VELOCITY VARIATIONS

(One Mile Intervals)

FLIGHT DESCRIPTION	MILES	$(w')^2$	2 MI.AVG.	4 MI. AVG.
Round Hill	1	16.13		
300'-180°	2	12.21	14.17	
	3	8.69		12.29
	4	12.13	10.41	
	5	9.76		10.54
	6	11.57	10.67	
	7	13.41		11.03
	8	9.34	11.38	
	Last Mile	4.13		
Round Hill	1	12.96		
300'-150°	2	1.84	7.40	
	3	2.19		5.18
	4	3.72	2.96	
	5	3.54		2.54
	6	0.69	2.12	
	7	2.35		1.89
	8	0.98	1.67	
	9	0.72		
Peekskill	1	39.54		
1000'-135°	2	34.15	36.85	
	3	39.00		30.87
	4	10.76	24.88	
	5	10.08		18.97
	6	16.01	13.05	
	7	14.39		16.16
	8	24.12	19.26	
Peekskill	1	8.47		
1000'-310°	2	22.67	15.57	
	3	10.32		11.89
	4	6.08	8.20	
	5	9.24		8.16
	6	6.99	8.12	
	7	11.62		8.45
	8	5.92	8.77	
	9	26.29		14.94
	10	15.93	21.11	
	11	18.98		17.61
	12	9.24	14.11	
	13	13.83		

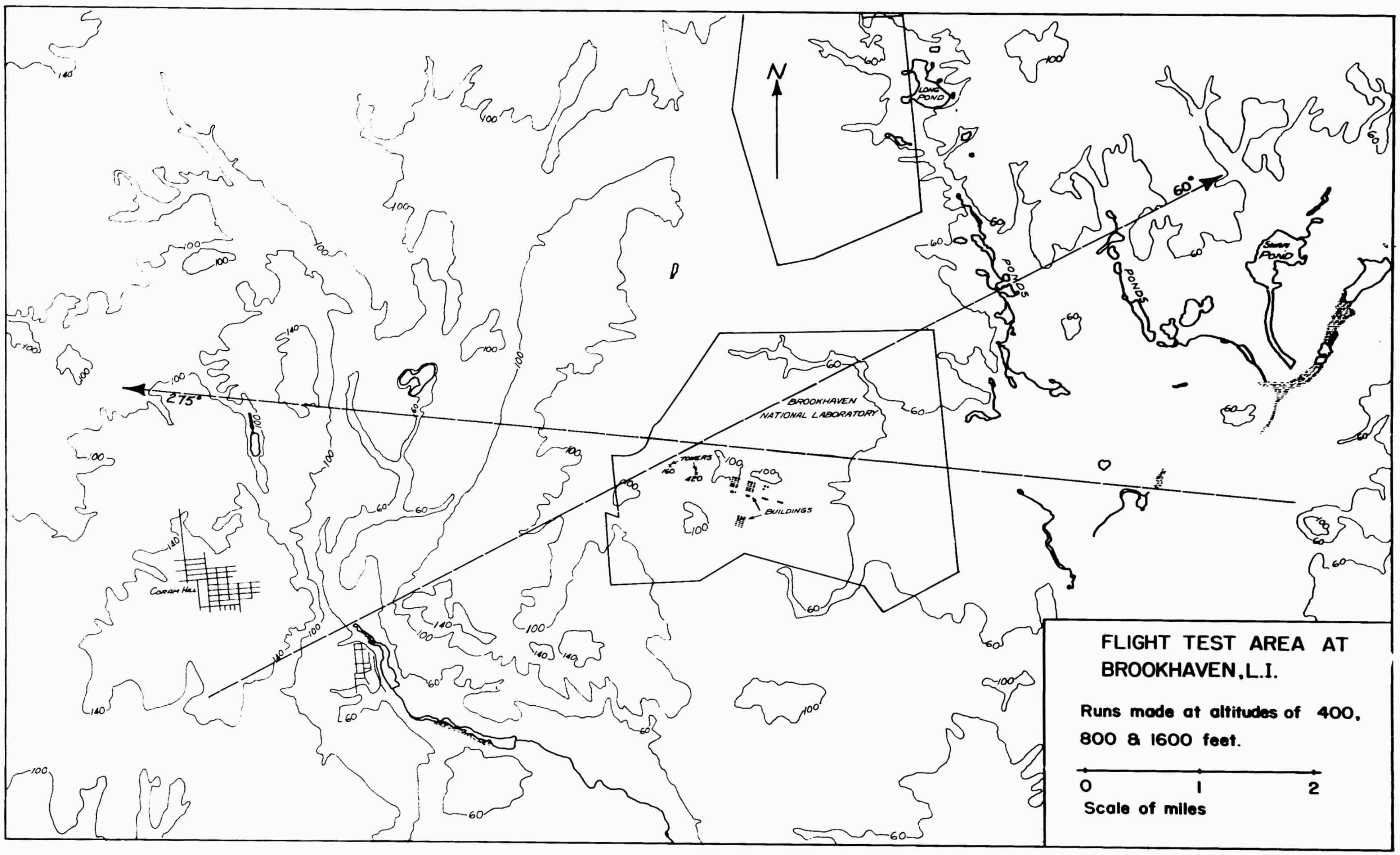
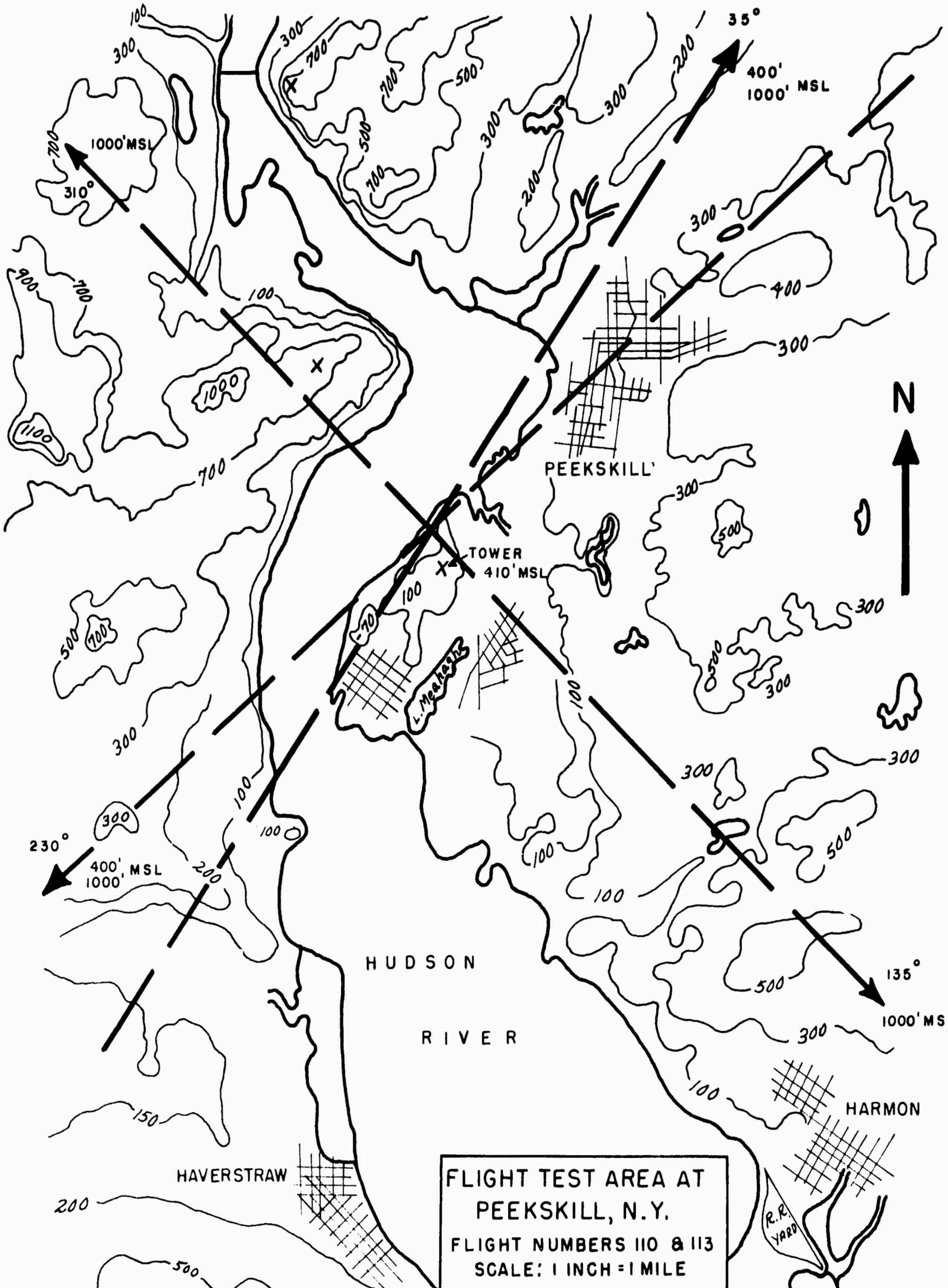


Figure 1. Flight test area at Brookhaven, L. I.



FLIGHT TEST AREA AT PEEKSKILL, N.Y.
FLIGHT NUMBERS 110 & 113
SCALE: 1 INCH = 1 MILE
N
35°
400'
1000' MSL
310°
1000'MSL
230°
400'
1000' MSL
135°
1000'MSL
TOWER
410'MSL
PEEKSKILL
HUDSON
RIVER
HAVERSTRAW
HARMON
R.R. YARD
L. Meahagh

Figure 2.

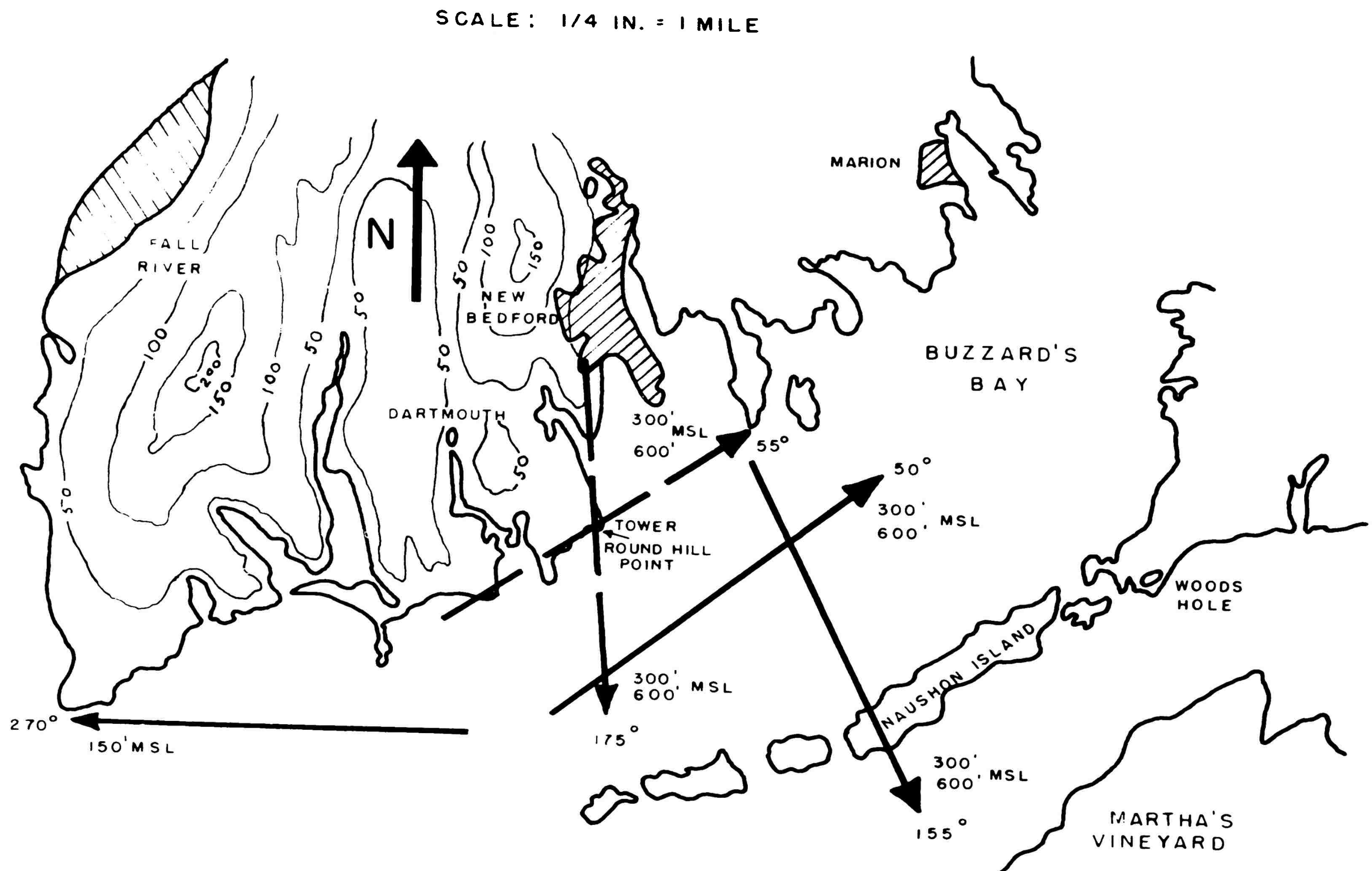


Figure 3. Flight test area at Round Hill, Mass.

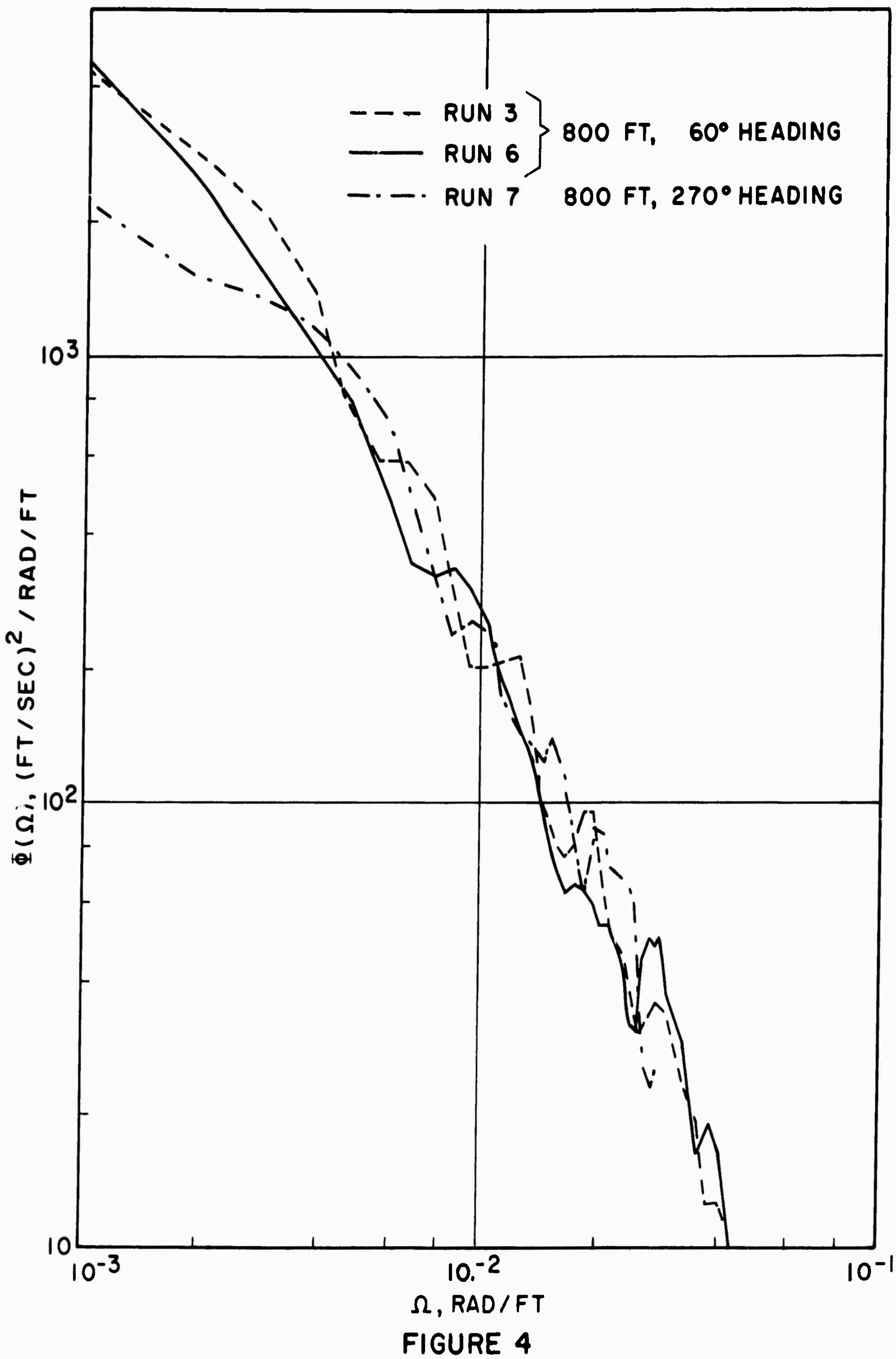


FIGURE 4

VERTICAL SPECTRA AT BROOKHAVEN SHOWING REPEAT RUNS AND EFFECT OF HEADING CHANGE ON APRIL 24, 1956.

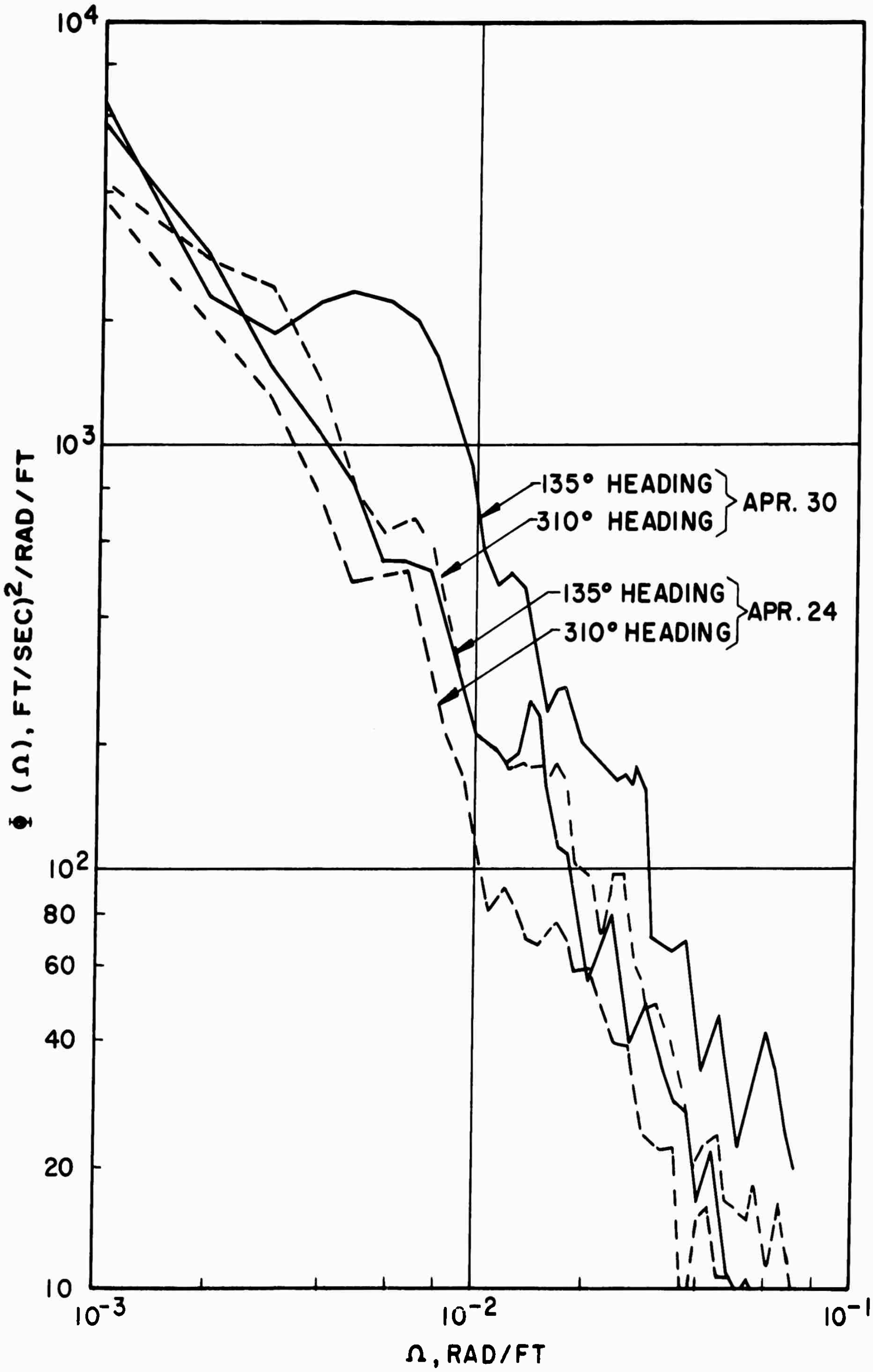


FIGURE 5

VARIABILITY OF VERTICAL VELOCITY SPECTRA AT 1000 FT AT PEEKSKILL ON APRIL 24 AND APRIL 30.

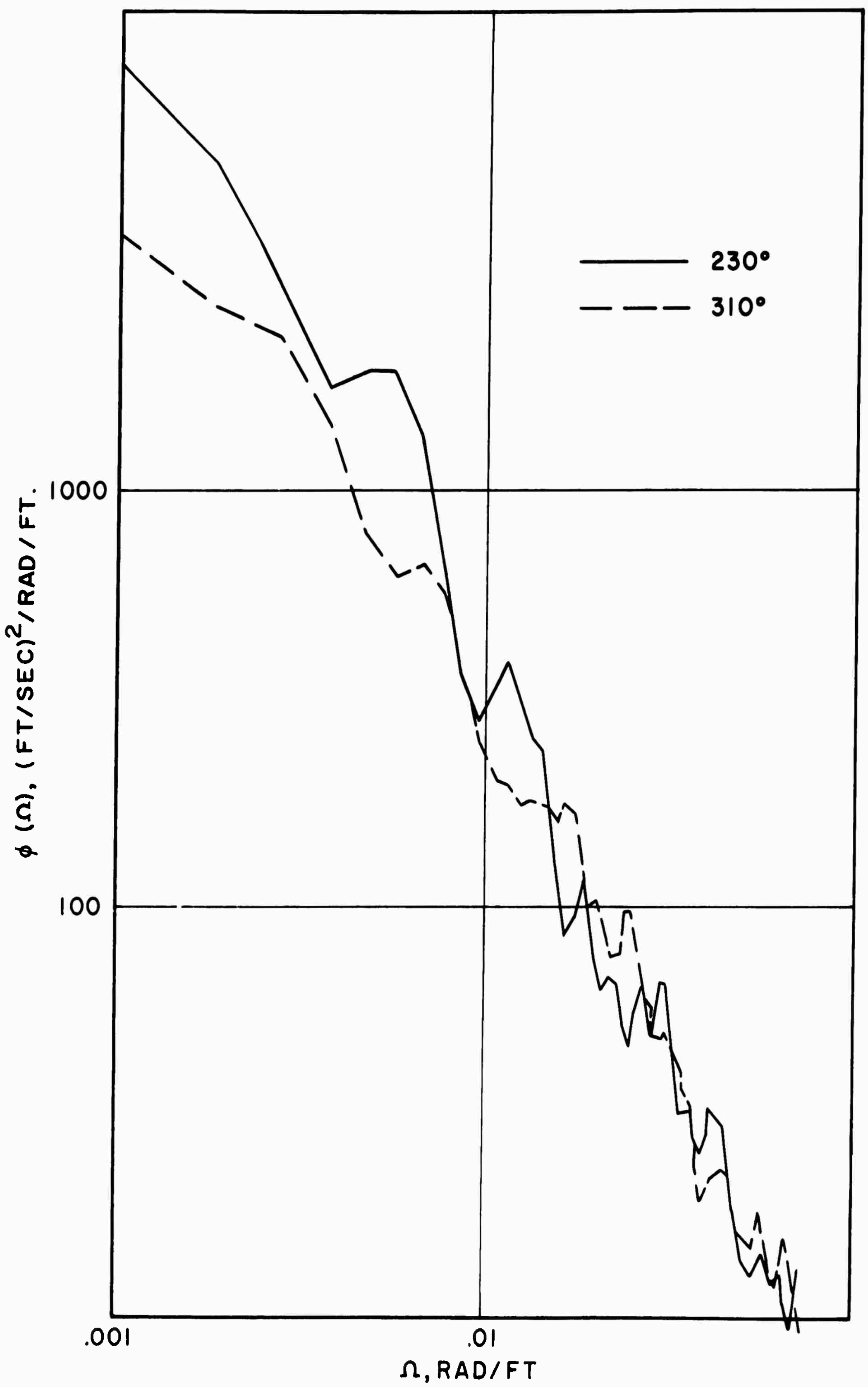


FIGURE 6

EFFECT OF FLIGHT PATH ORIENTATION
ON VERTICAL VELOCITY SPECTRA AT PEEKSKILL (APRIL 30)

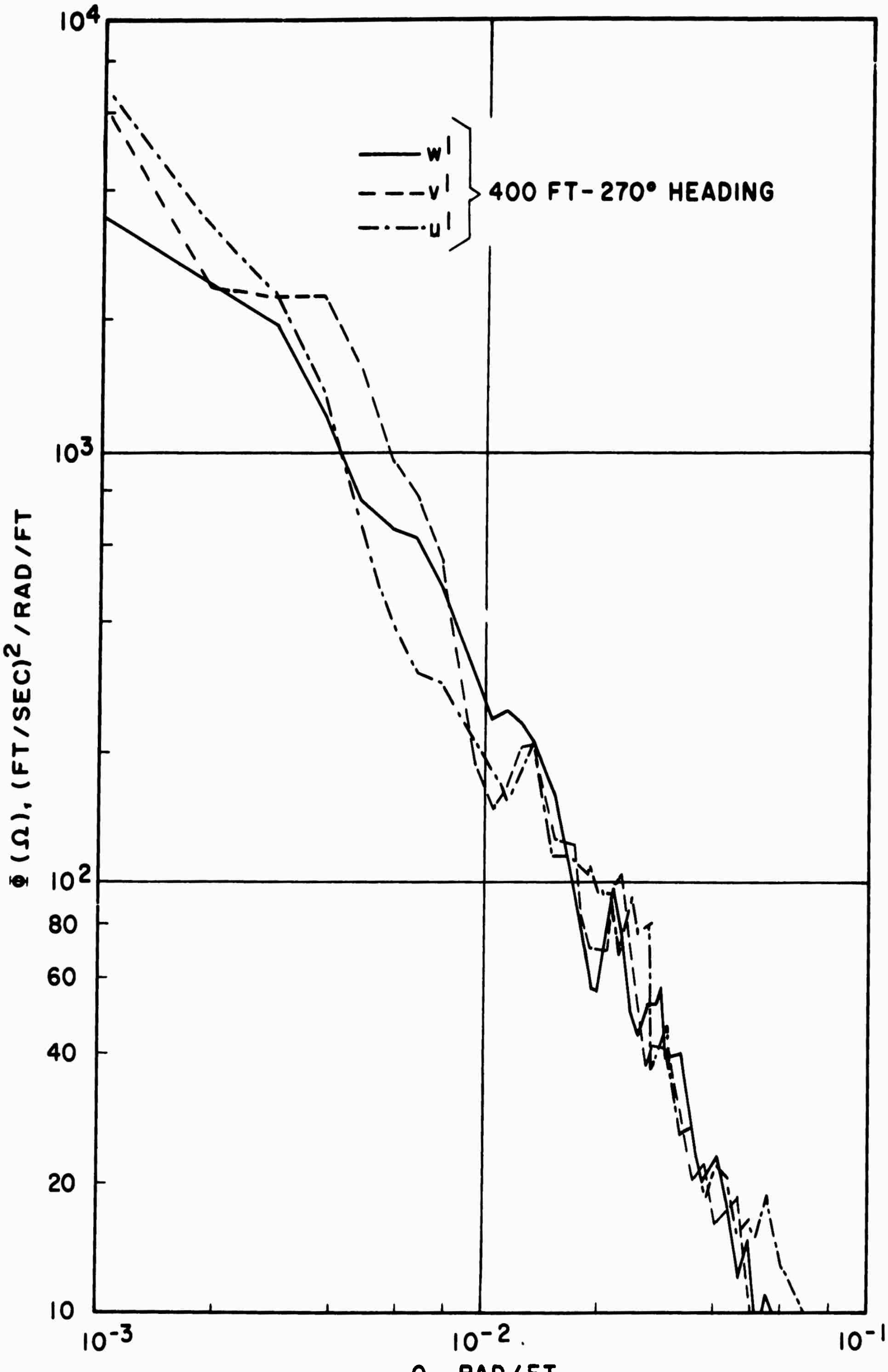


FIGURE 7

AIRPLANE VELOCITY SPECTRA AT BROOKHAVEN, APRIL 24, 1956

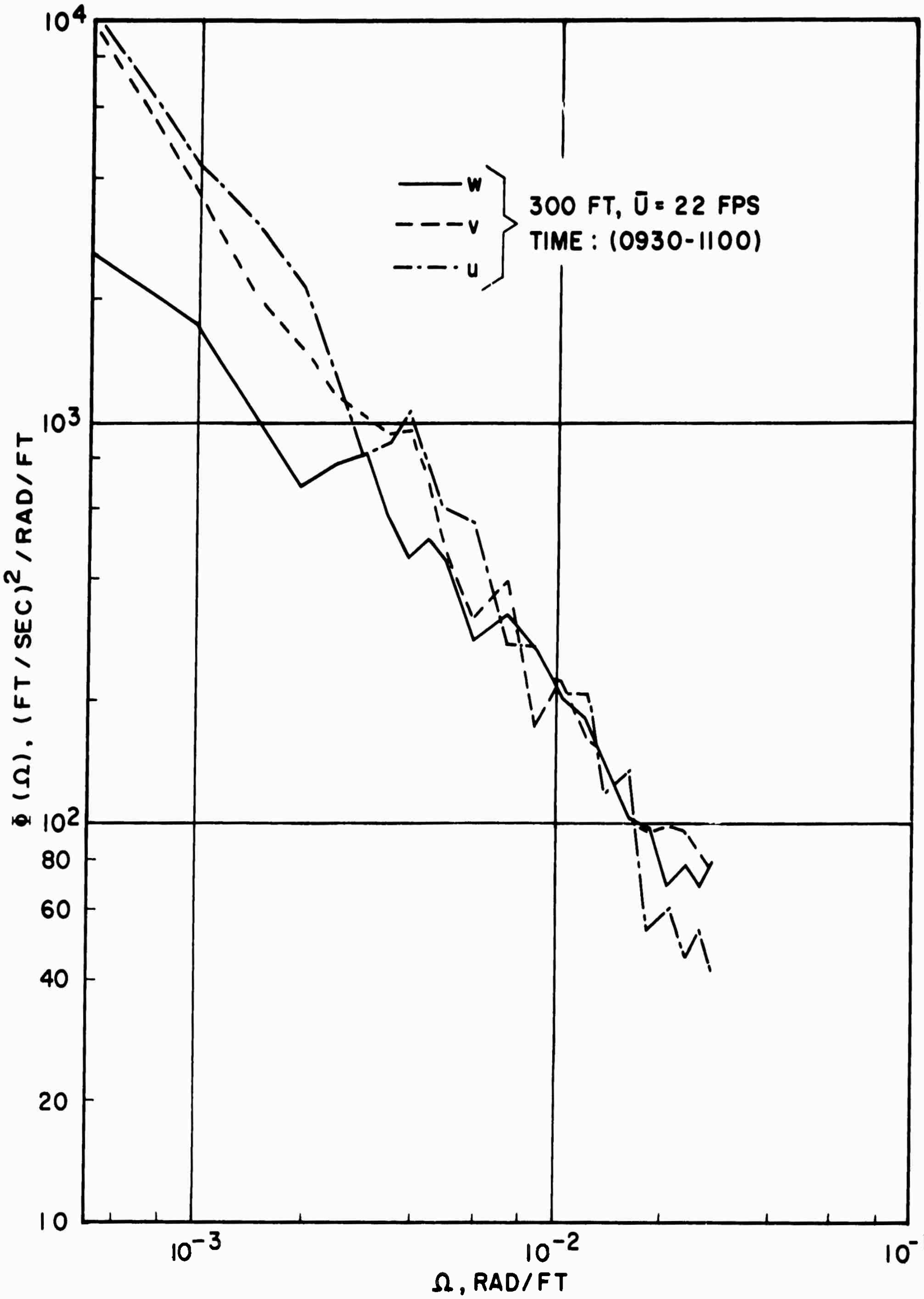


FIGURE 8
TOWER VELOCITY SPECTRA AT BROOKHAVEN
ON APRIL 24, 1956

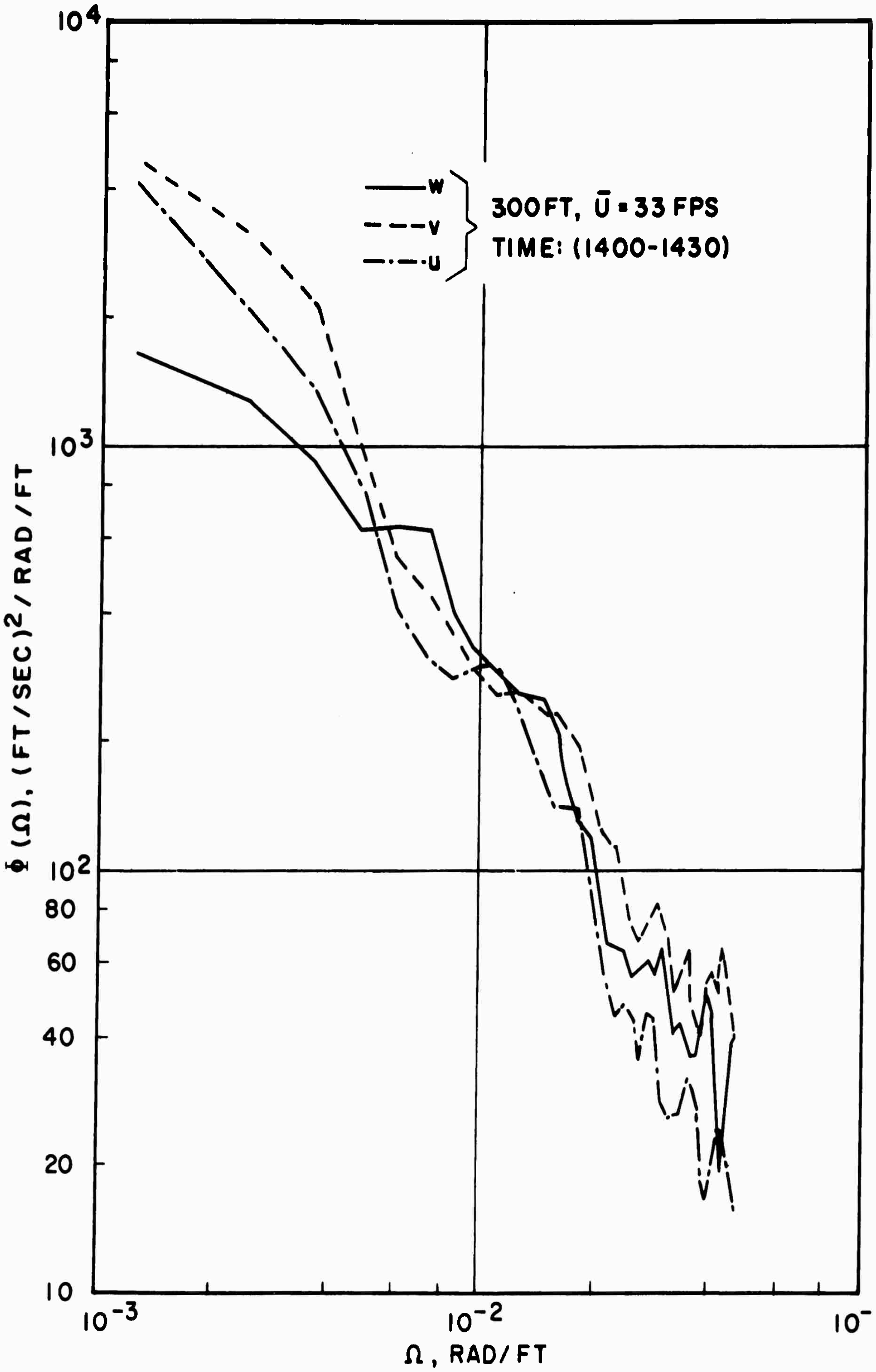


FIGURE 9

TOWER VELOCITY SPECTRA AT PEEKSKILL, APRIL 30, 1956

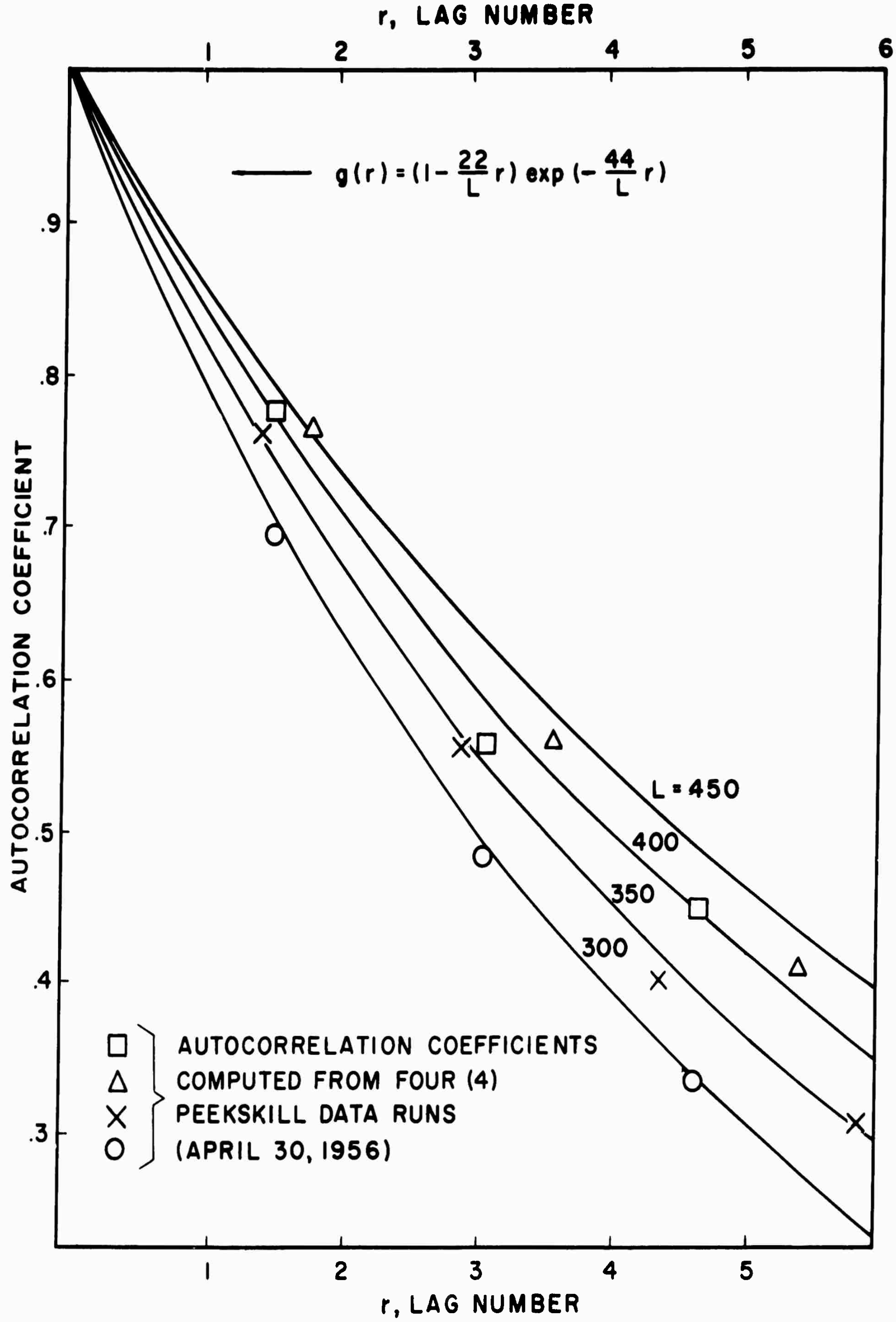


FIGURE 10

PEEKSKILL TOWER DATA COMPARED WITH CORRELATION FUNCTIONS CORRESPONDING TO A -2 POWER LAW

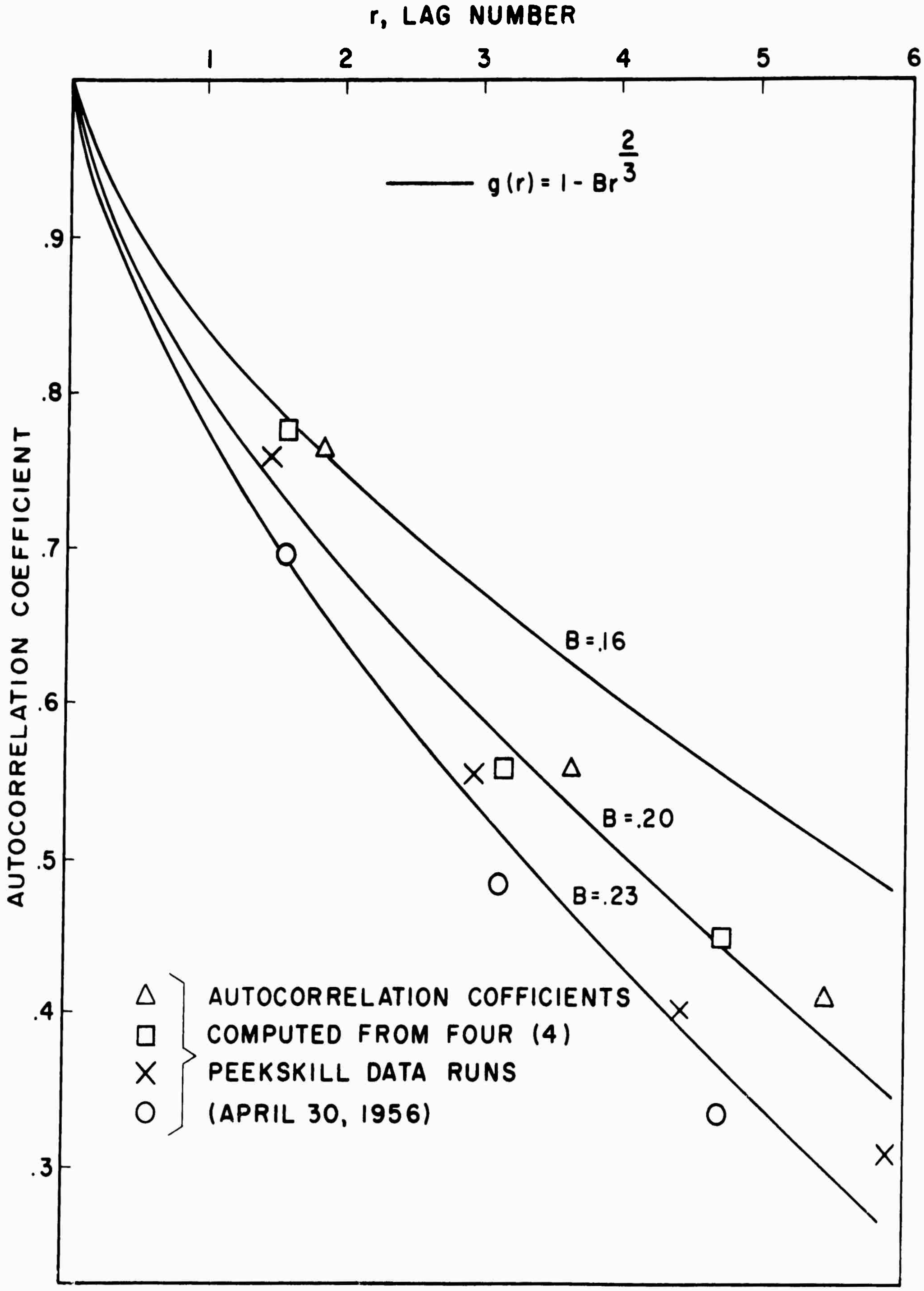


FIGURE II

PEEKSKILL TOWER DATA COMPARED WITH CORRELATION FUNCTIONS CORRESPONDING TO $-\frac{5}{3}$ POWER LAW.

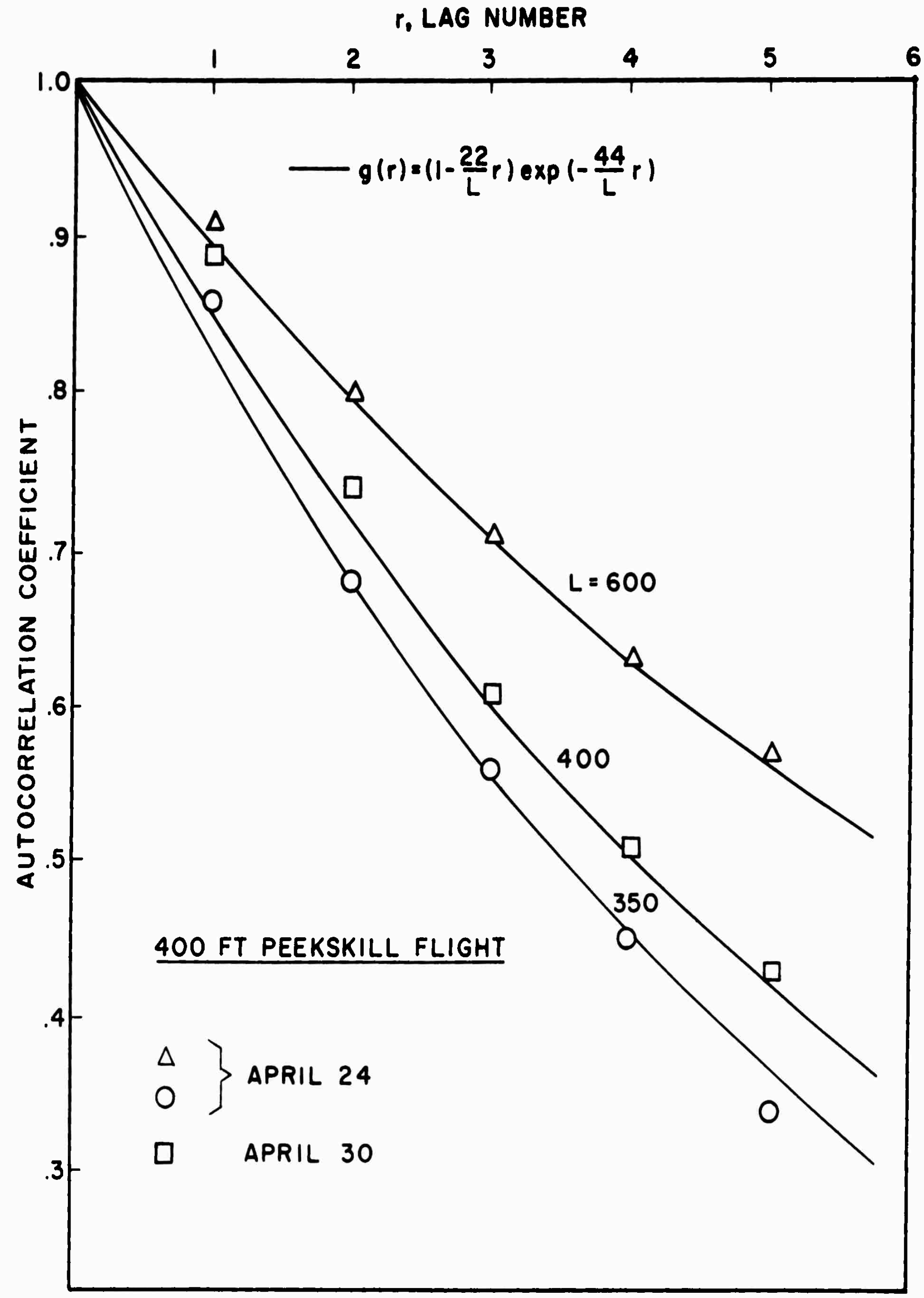


FIGURE 12

PEEKSKILL AIRPLANE DATA COMPARED WITH CORRELATION FUNCTIONS CORRESPONDING TO A -2 POWER LAW.

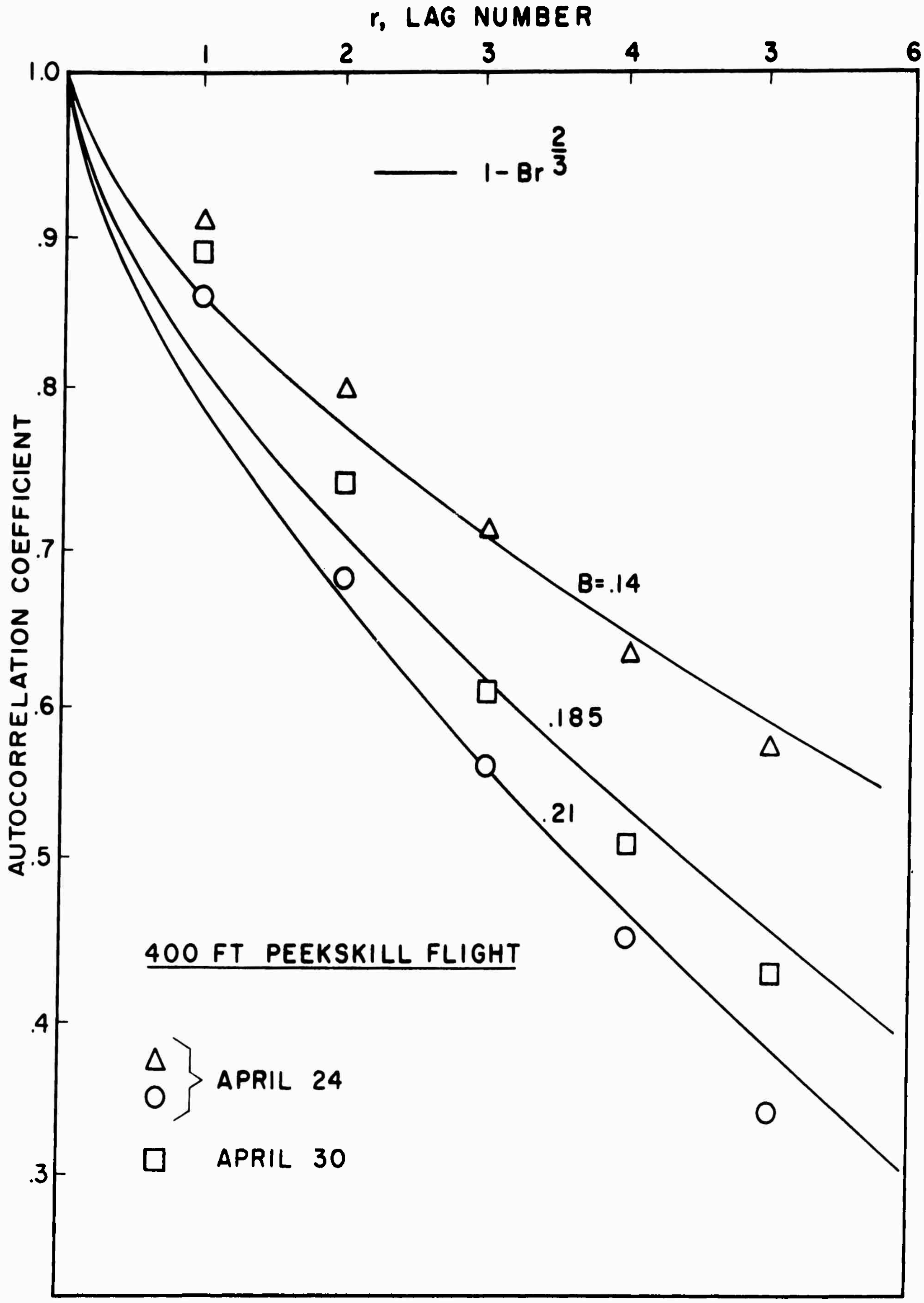


FIGURE 13

PEEKSKILL AIRPLANE DATA COMPARED WITH CORRELATION FUNCTIONS CORRESPONDING TO A $-\frac{5}{3}$ POWER LAW.

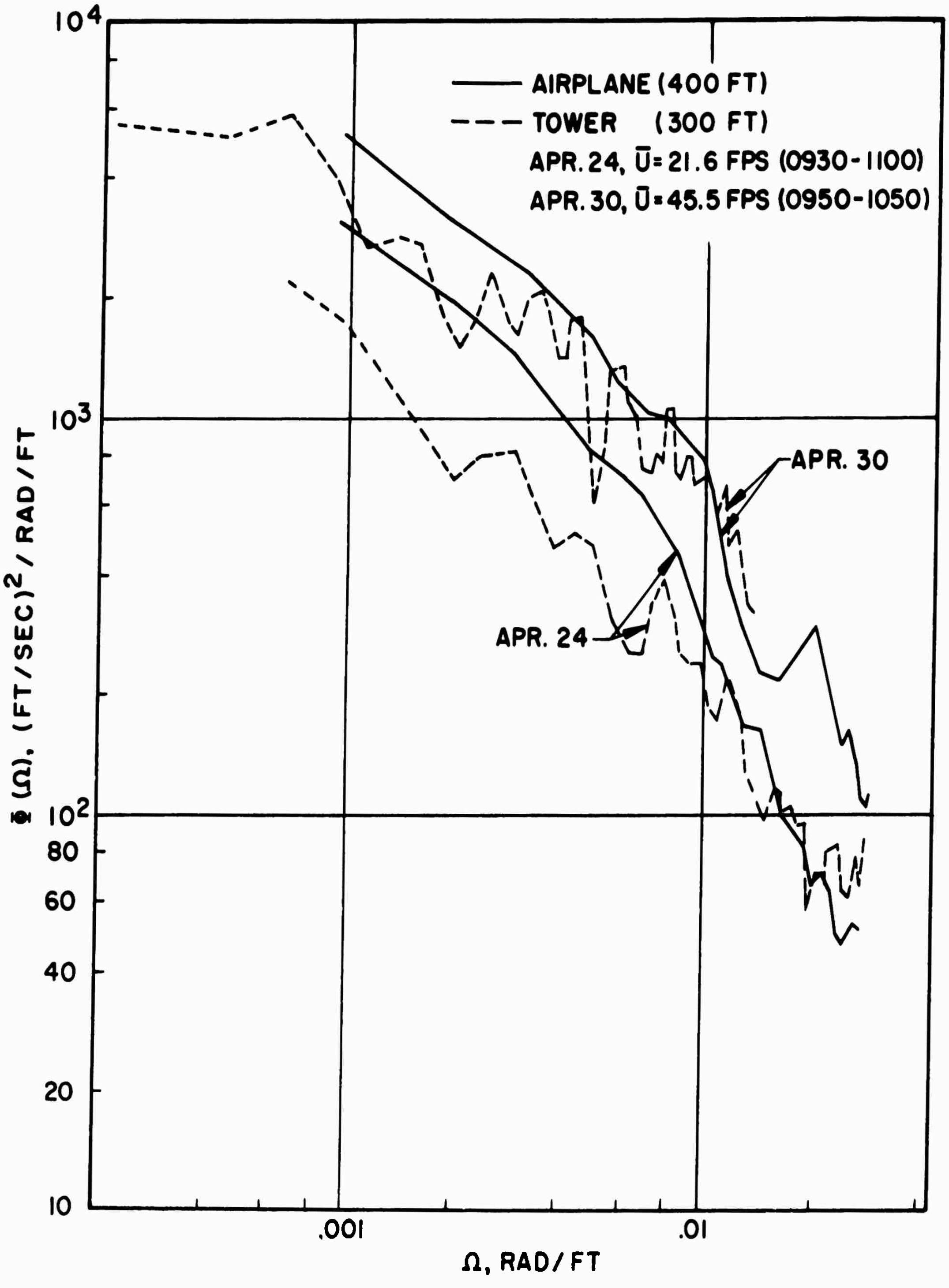


FIGURE 14

COMPARISON OF AIRPLANE-TOWER VELOCITY SPECTRA AT BROOKHAVEN ON APRIL 24 AND APRIL 30.
(A) VERTICAL VELOCITY SPECTRA.

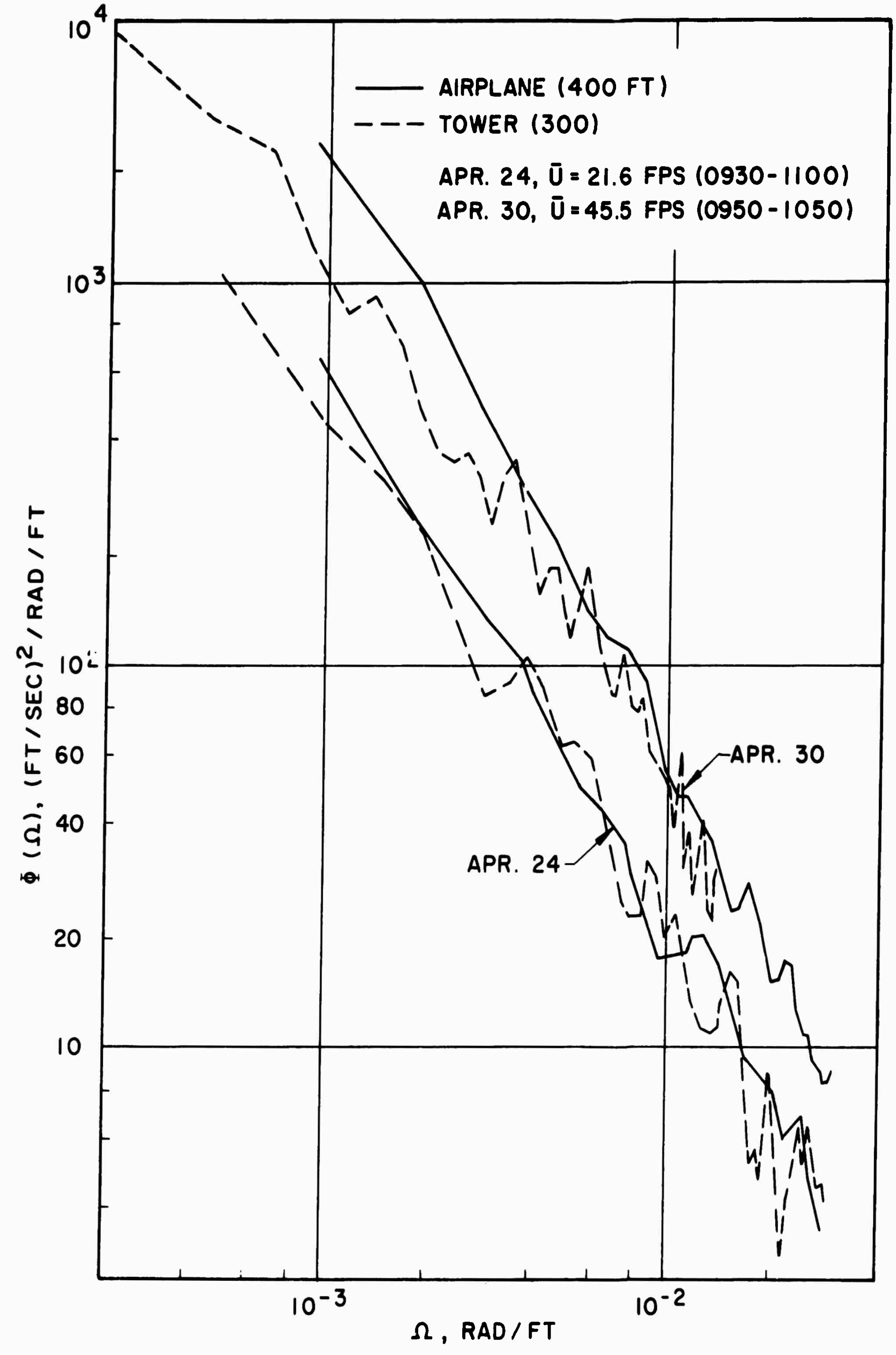


FIGURE 14

(B) HEAD-ON VELOCITY SPECTRA

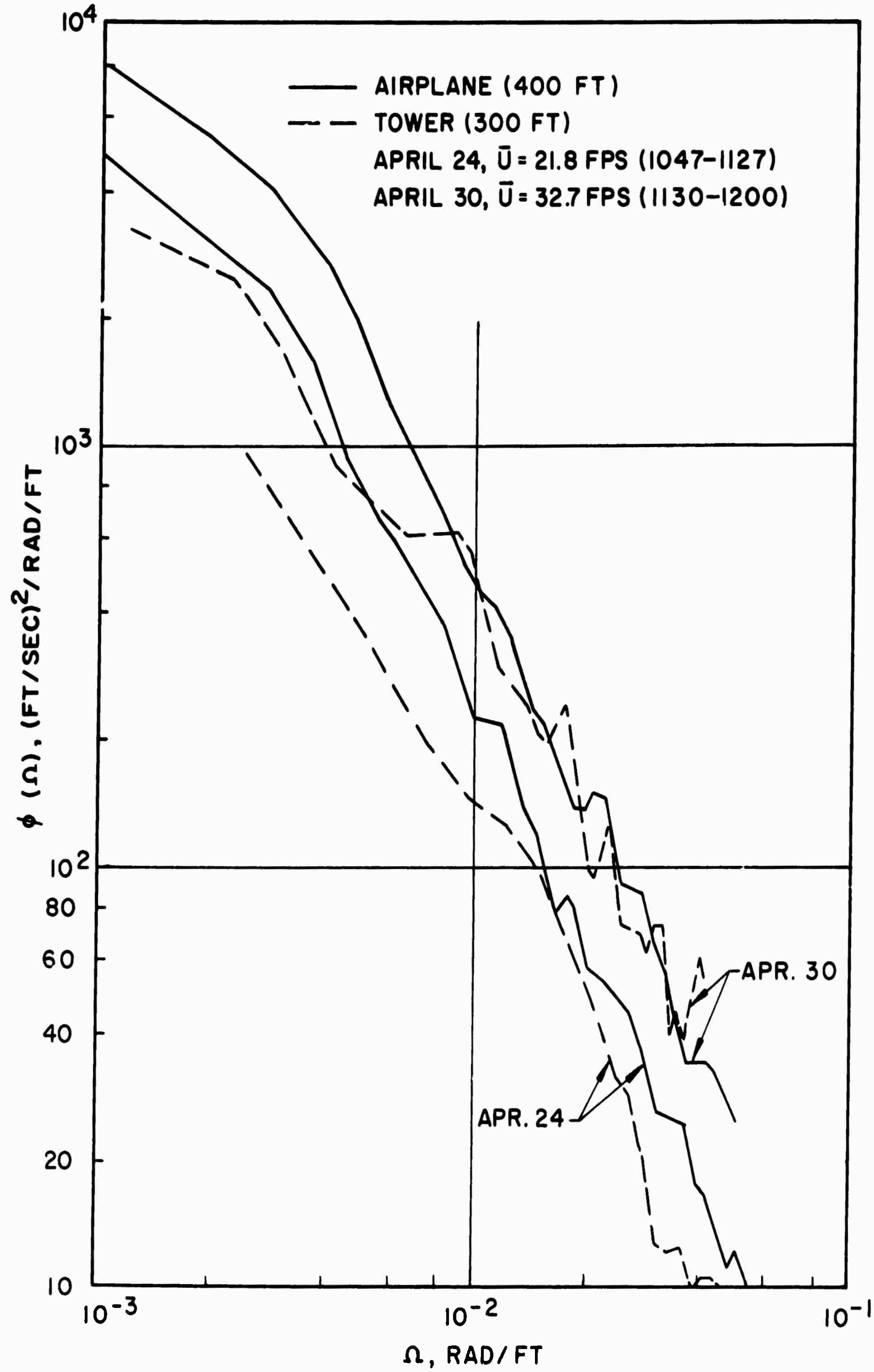


FIGURE 15

COMPARISON OF AIRPLANE-TOWER VELOCITY SPECTRA AT PEEKSKILL ON APRIL 24 AND APRIL 30.

(A) VERTICAL VELOCITY SPECTRA.

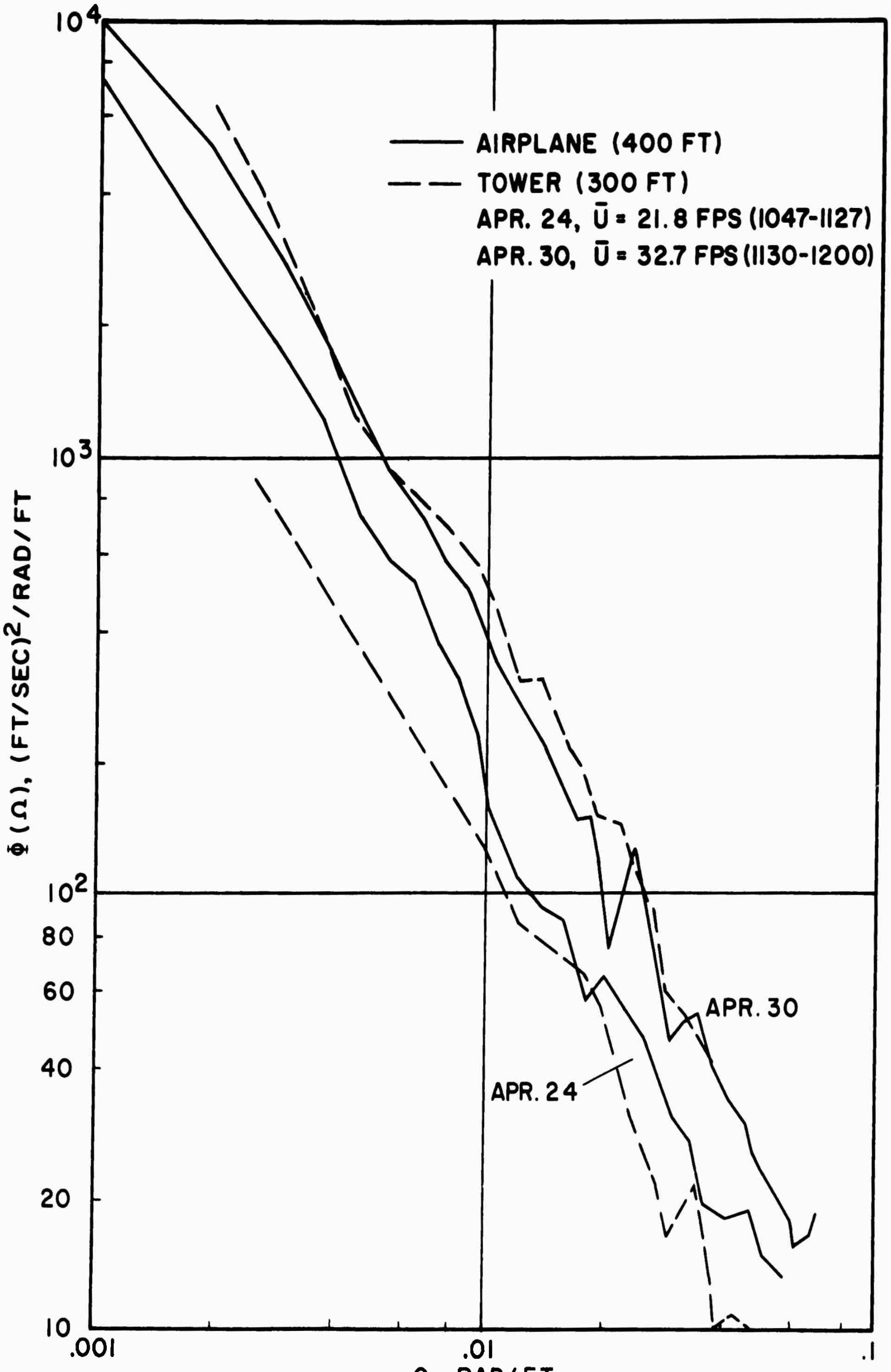


FIGURE 15

(B) HEAD-ON VELOCITY SPECTRA.

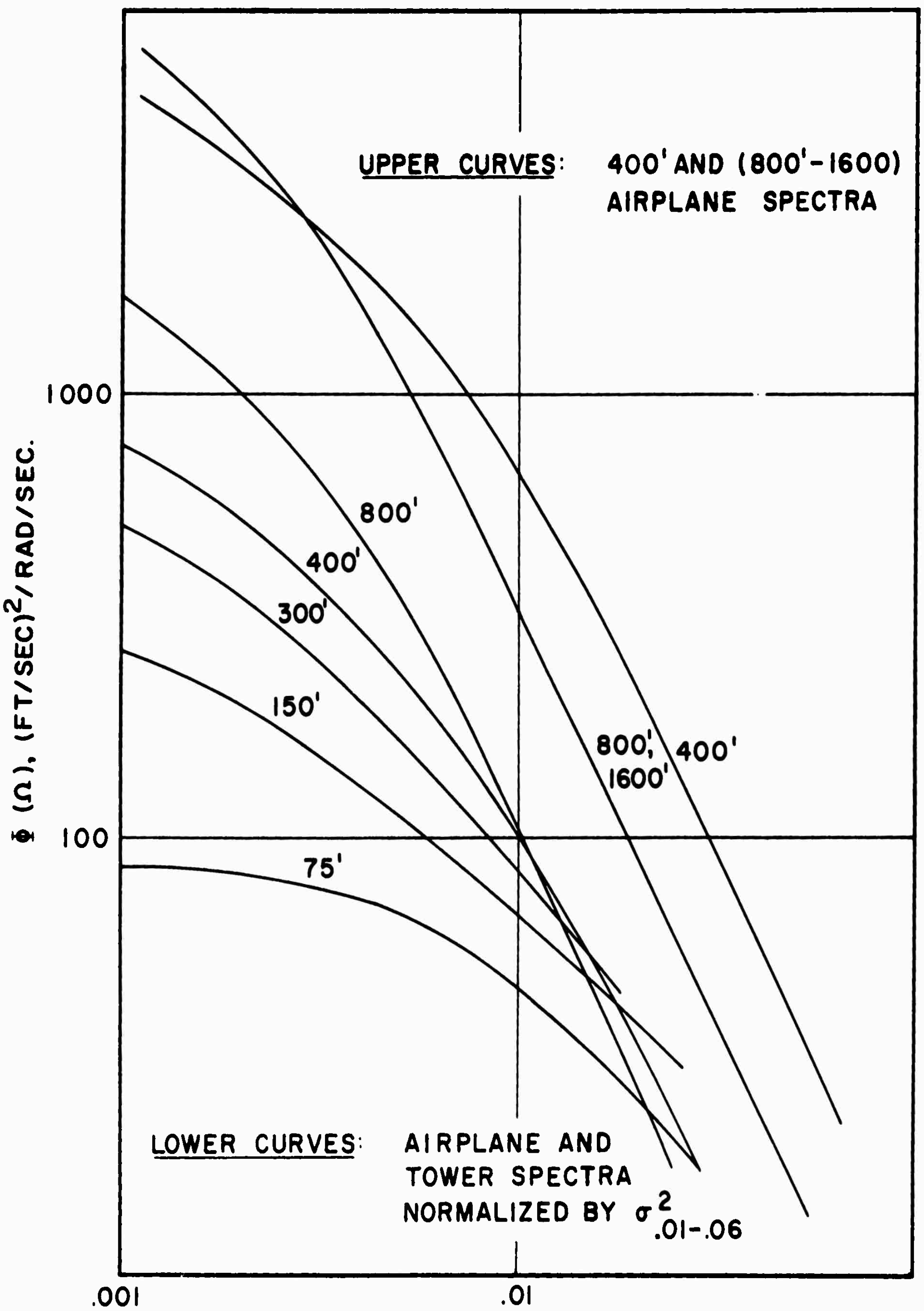


FIGURE 16

ALTITUDE VARIATION OF TOWER AND AIRPLANE FAIRED VERTICAL VELOCITY SPECTRA AT BROOKHAVEN, APRIL 30.

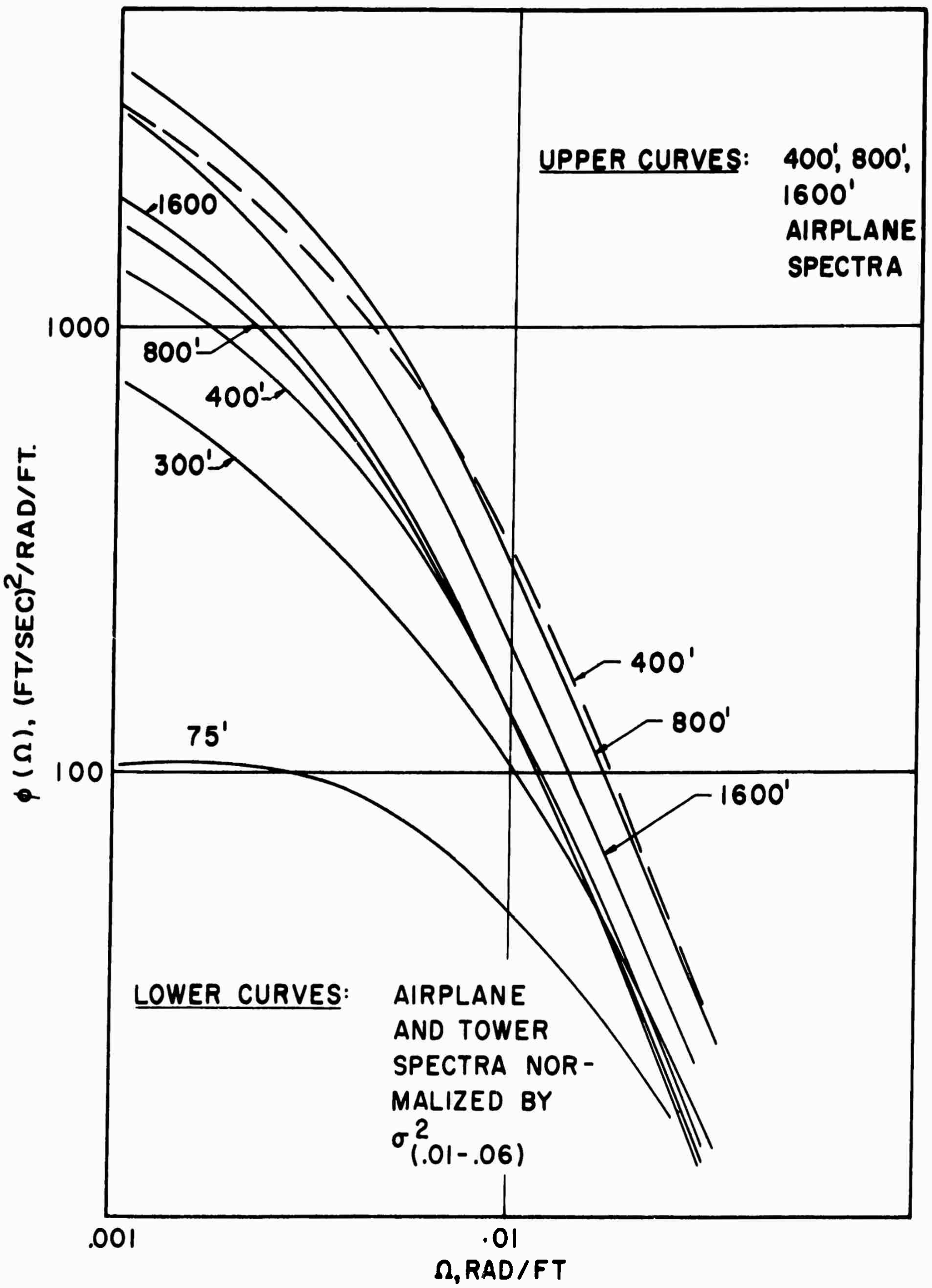


FIGURE 17

ALTITUDE VARIATION OF TOWER AND AIRPLANE FAIRED VERTICAL VELOCITY SPECTRA AT BROOKHAVEN, APRIL 24.

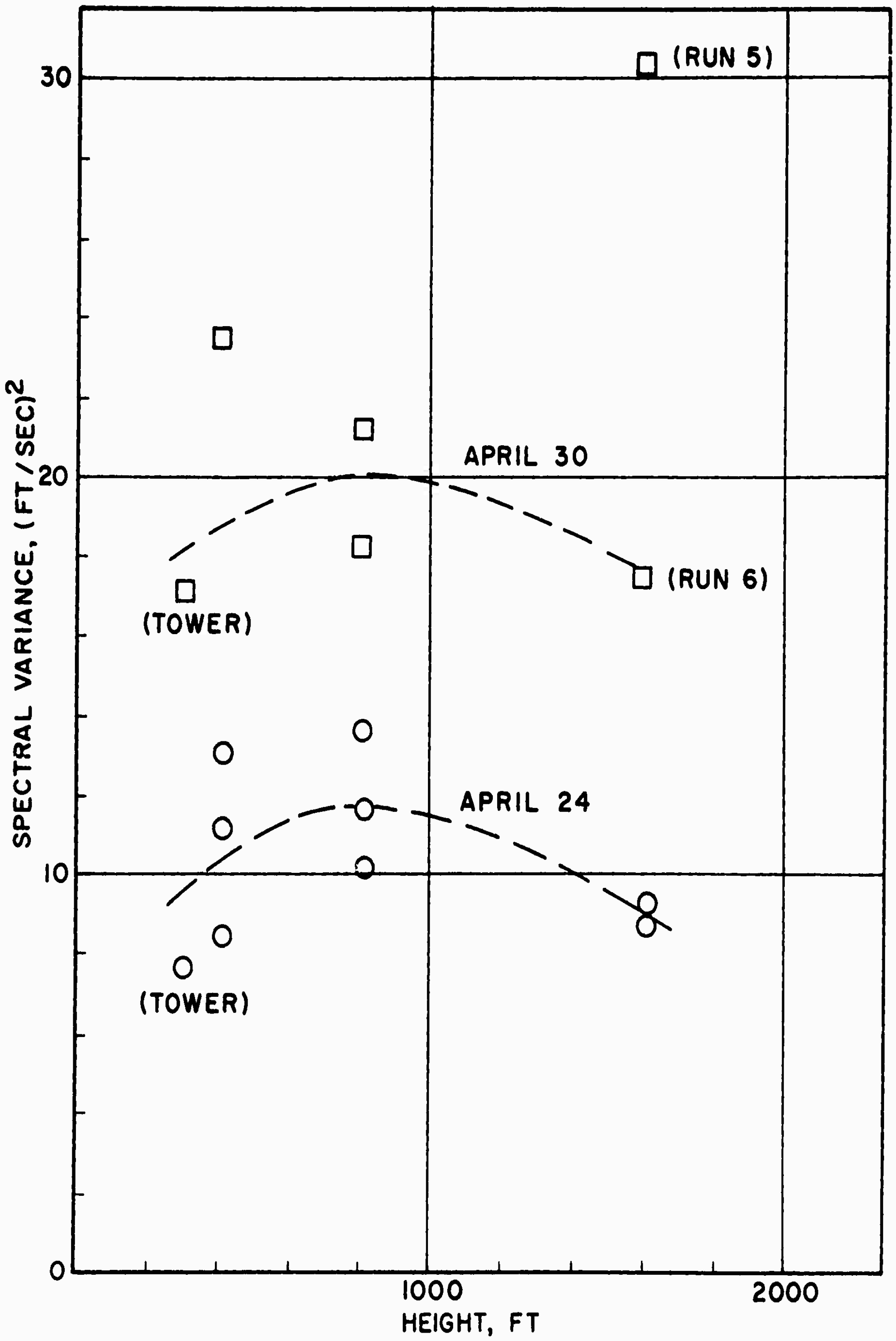


FIGURE 18
AIRPLANE AND TOWER SPECTRAL VARIANCES AT BROOKHAVEN APRIL 24 AND 30.

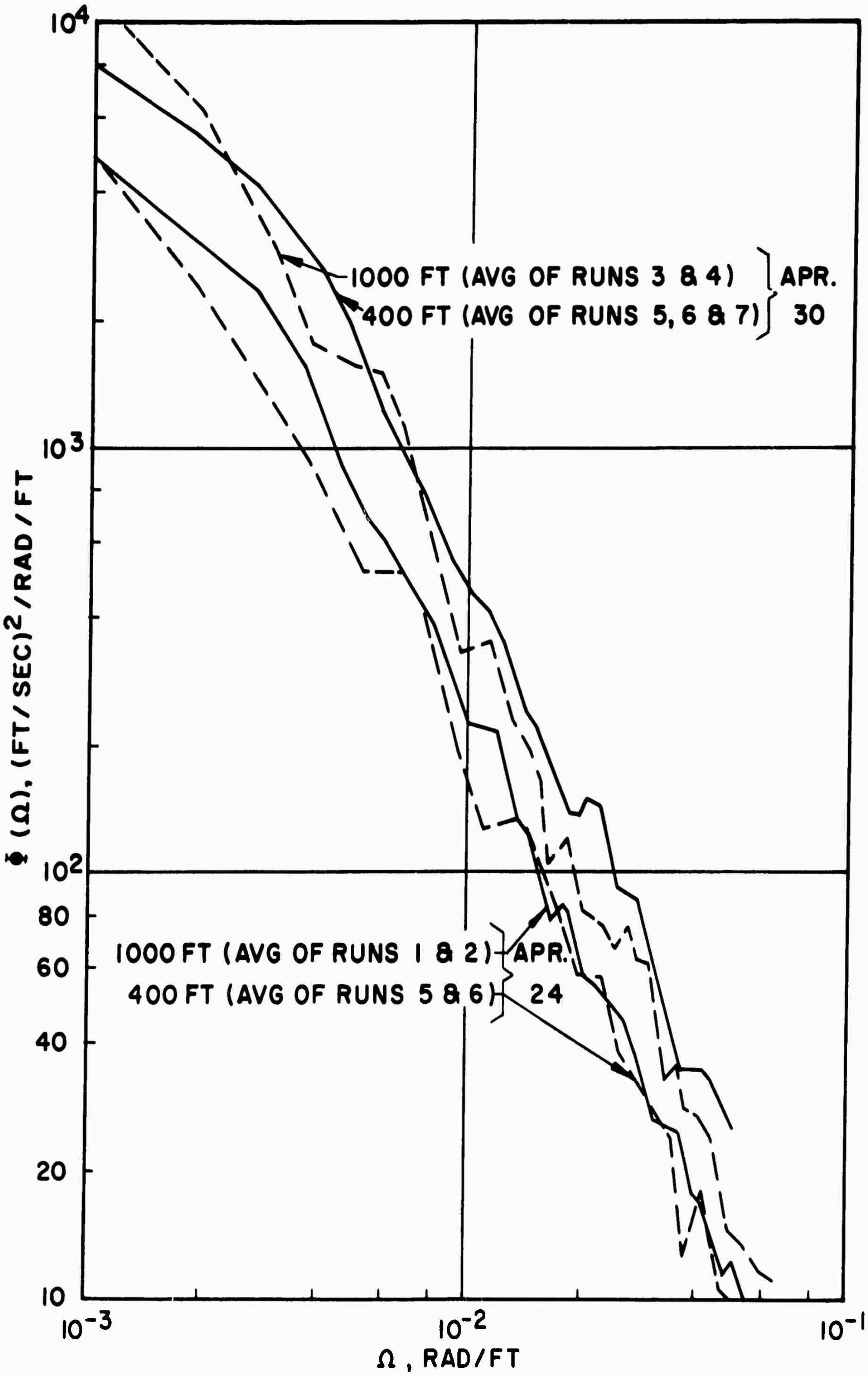


FIGURE 19

EFFECT OF ALTITUDE ON VERTICAL VELOCITY SPECTRA AT PEEKSKILL ON APRIL 24 AND APRIL 30.

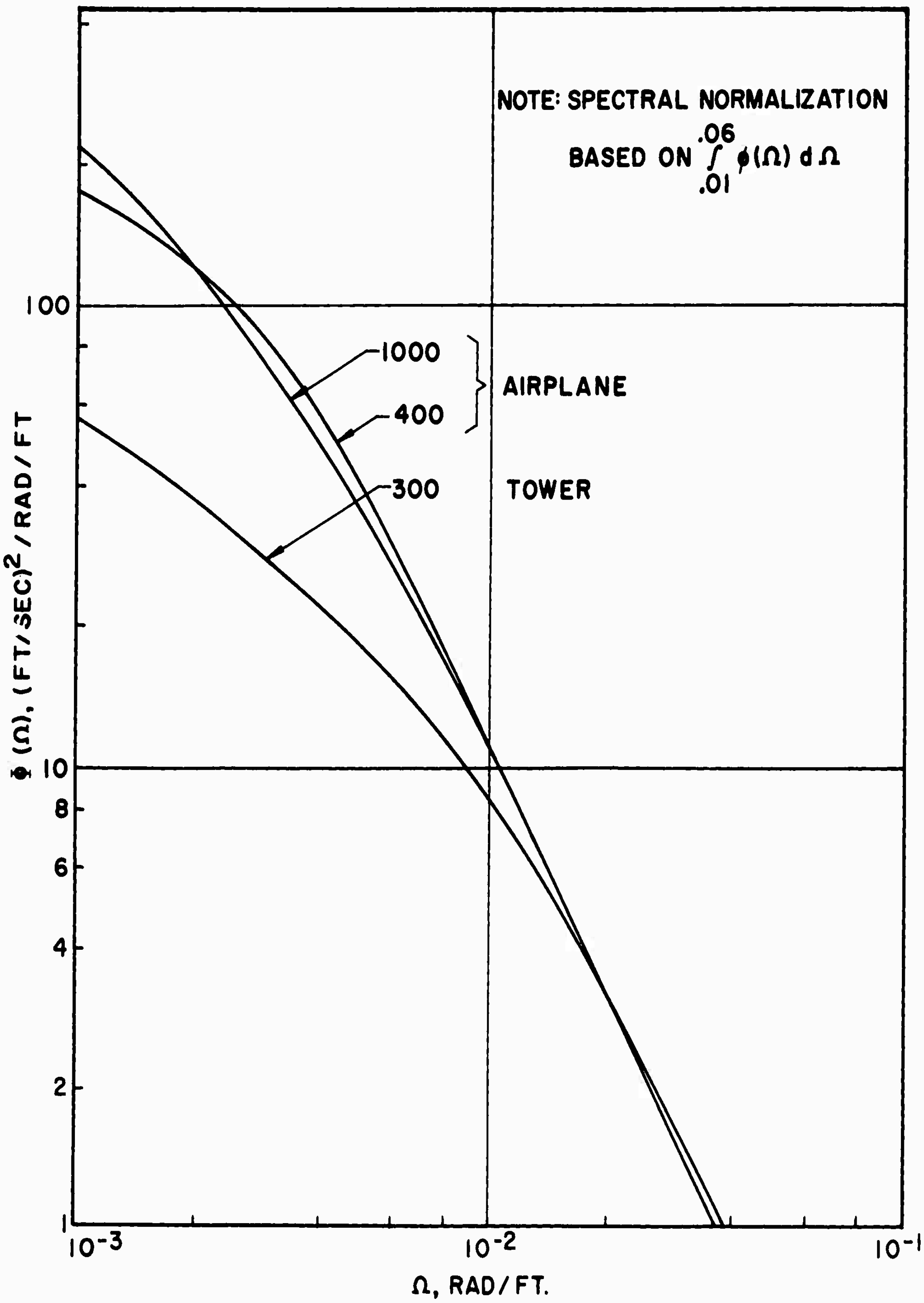


FIGURE 20

ALTITUDE VARIATION OF FAIRED AND NORMALIZED VERTICAL SPECTRA AT PEEKSKILL ON APR. 24-30.

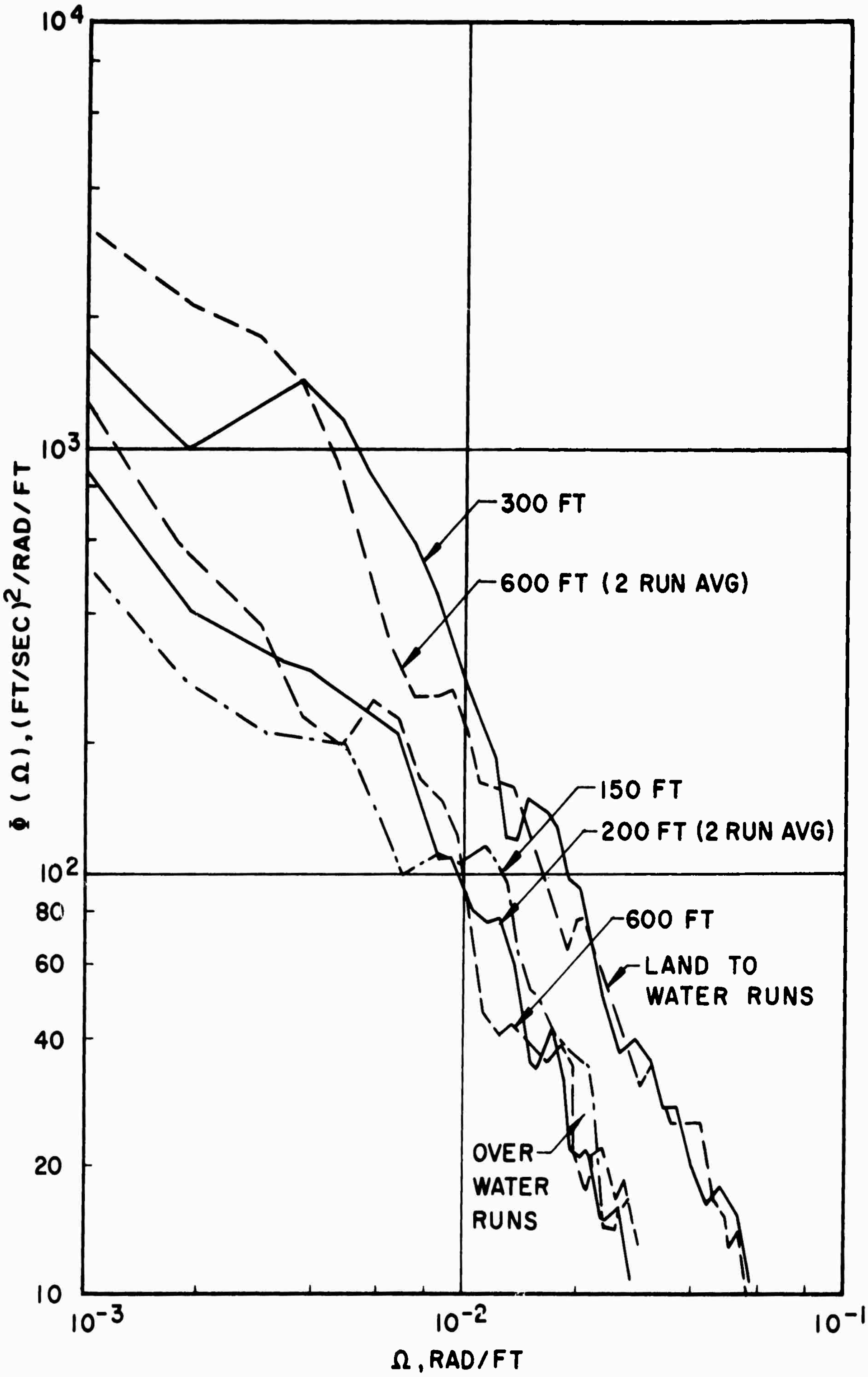


FIGURE 21
ALTITUDE VARIATION OF VERTICAL VELOCITY SPECTRA AT ROUND HILL ON APRIL 30.

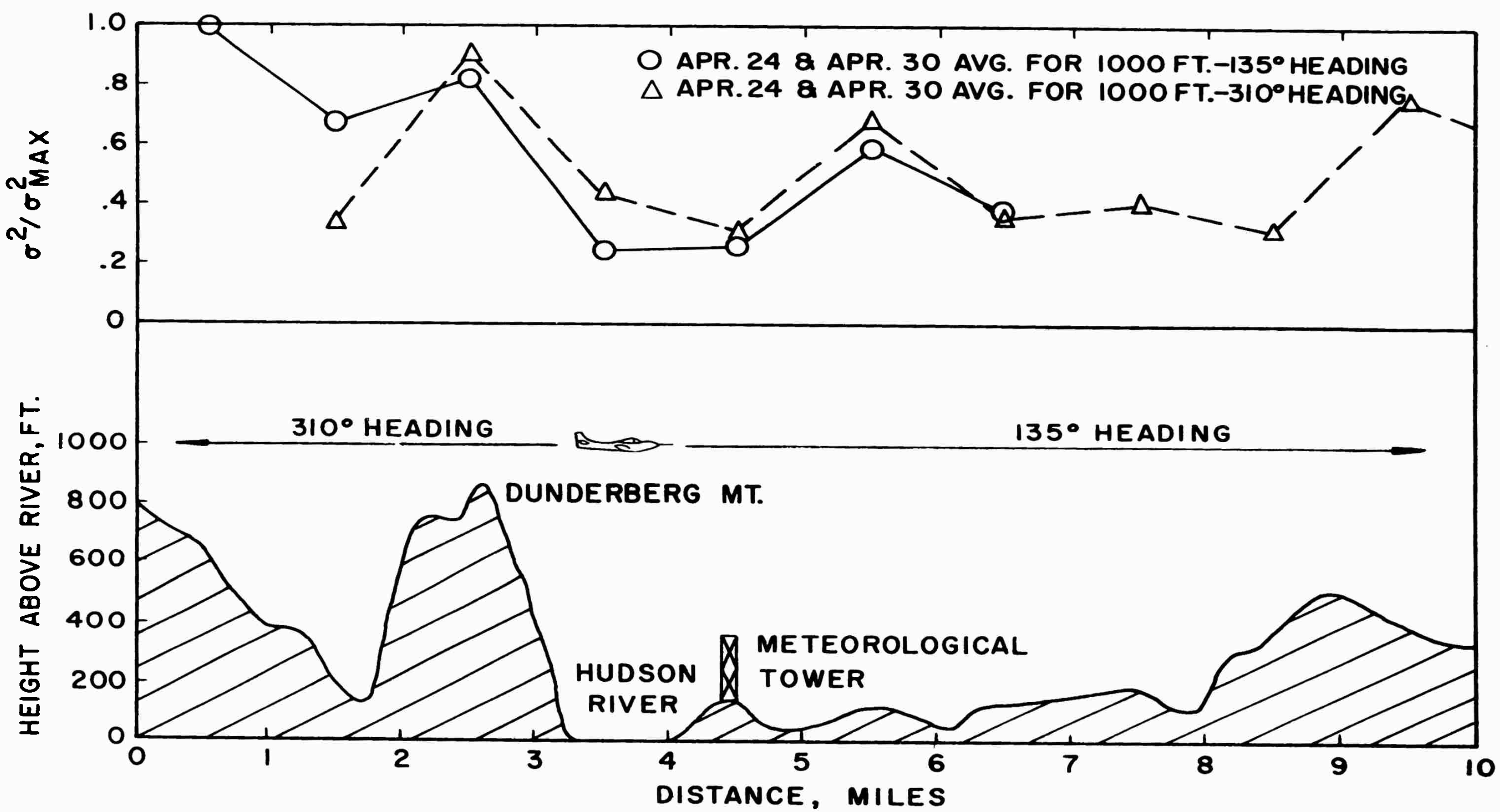


FIGURE 22

MEAN SQUARE VERTICAL FLUCTUATIONS COMPUTED AT ONE MILE INTERVALS FOR PEESKILL TOWER FLIGHTS

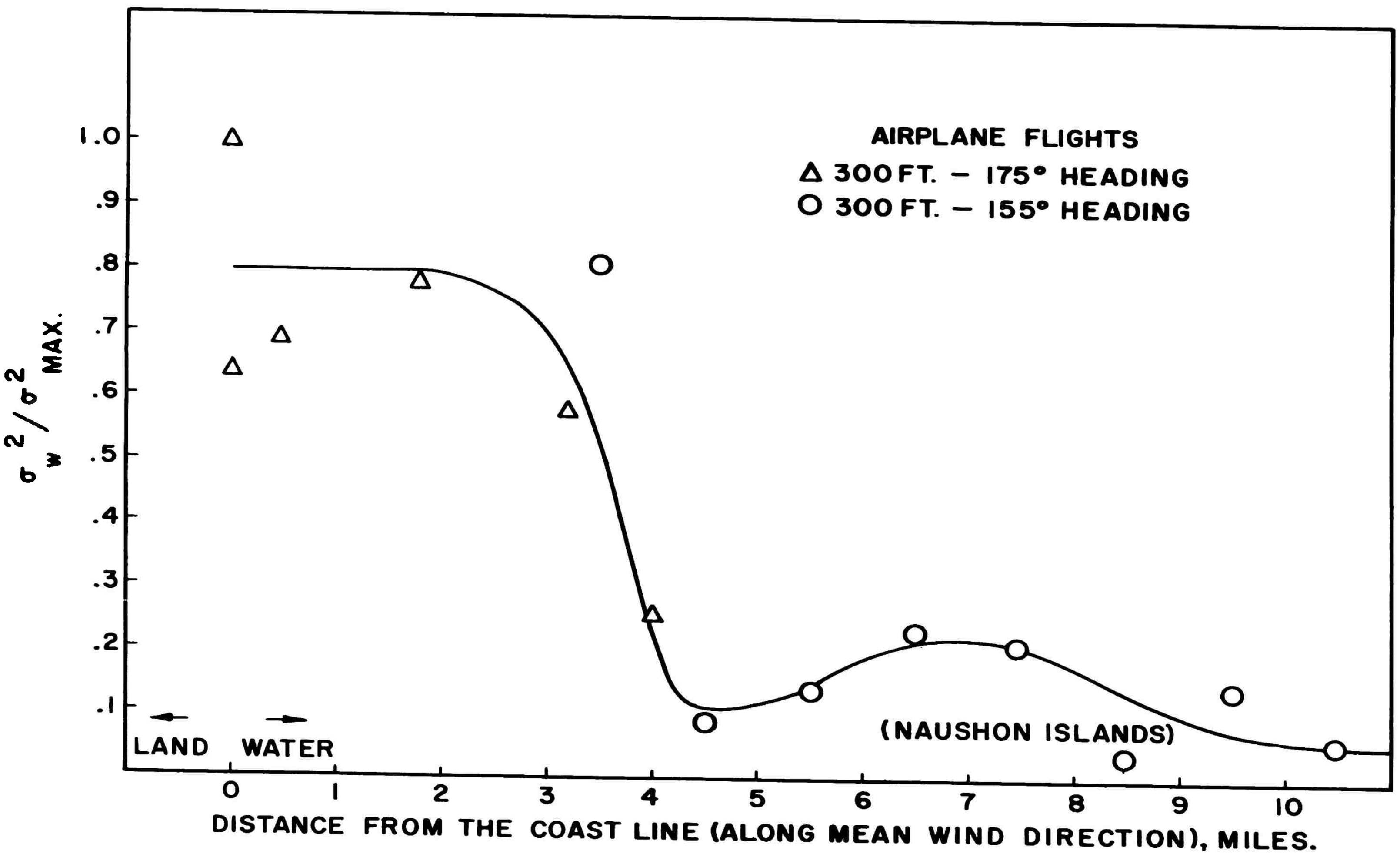


FIGURE 23
VERTICAL VELOCITY VARIANCES COMPUTED FOR ONE MILE SEGMENTS AT ROUND HILL ON APRIL 30.

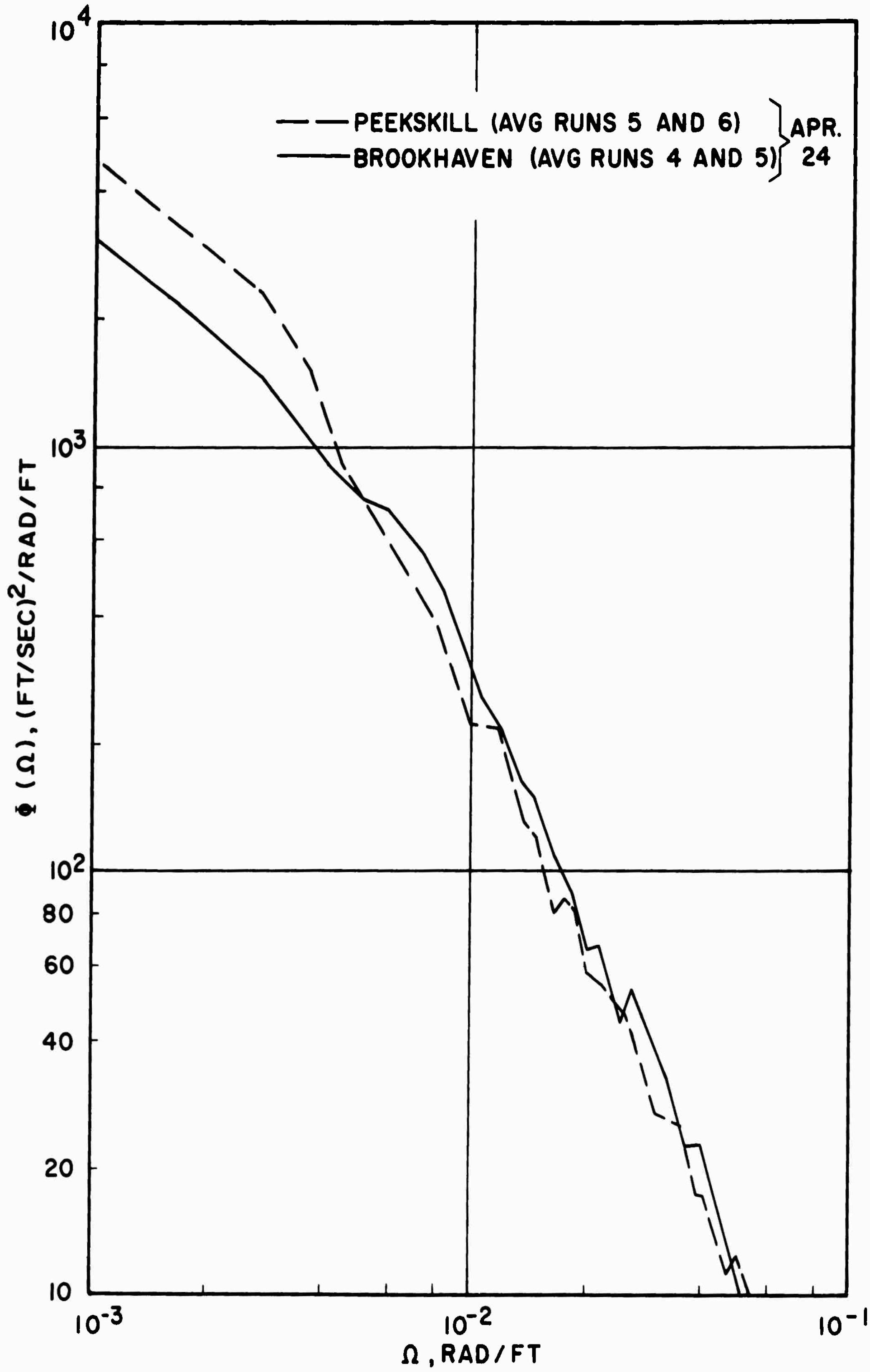


FIGURE 24

EFFECT OF TERRAIN ON VERTICAL VELOCITY SPECTRA AT 400 FT AT BROOKHAVEN AND PEEKSKILL, ON APRIL 24.

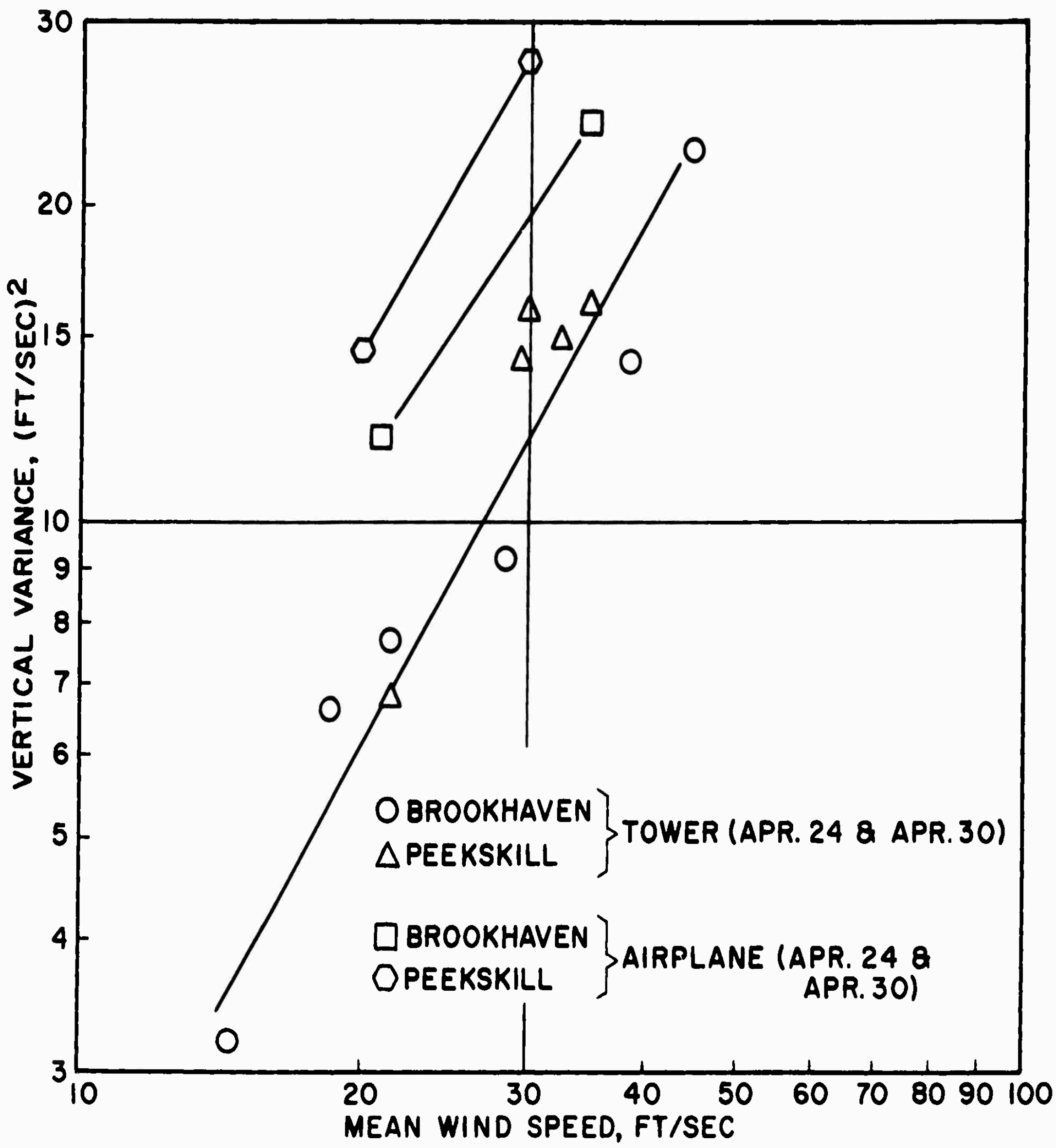


FIGURE 25

EFFECT OF MEAN WIND SPEED ON VERTICAL VARIANCES FOR TOWER AND AIRPLANE.

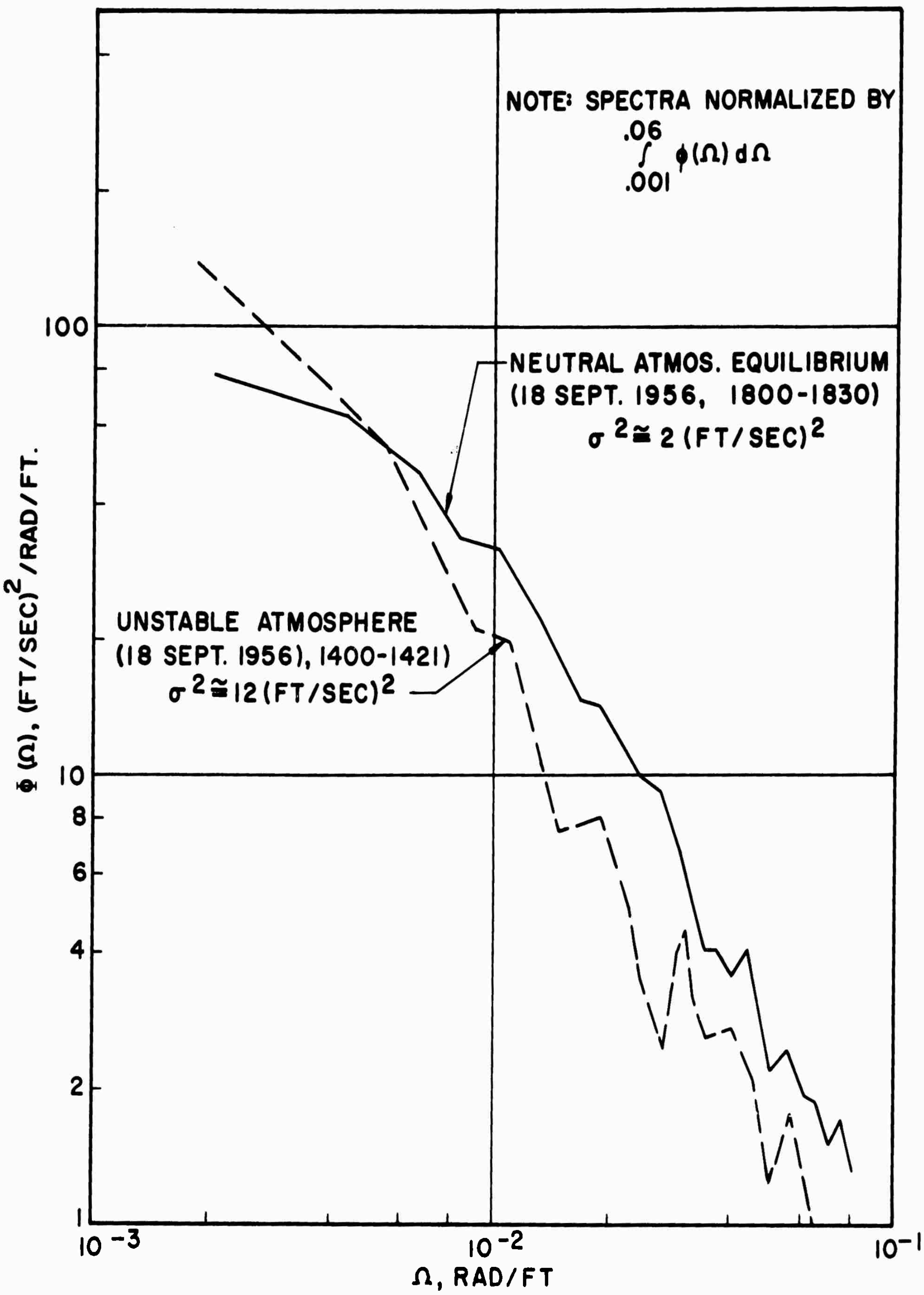


FIGURE 26

EFFECT OF ATMOSPHERIC STABILITY ON VERTICAL VELOCITY SPECTRA MEASURED AT THE PEEKSKILL TOWER.

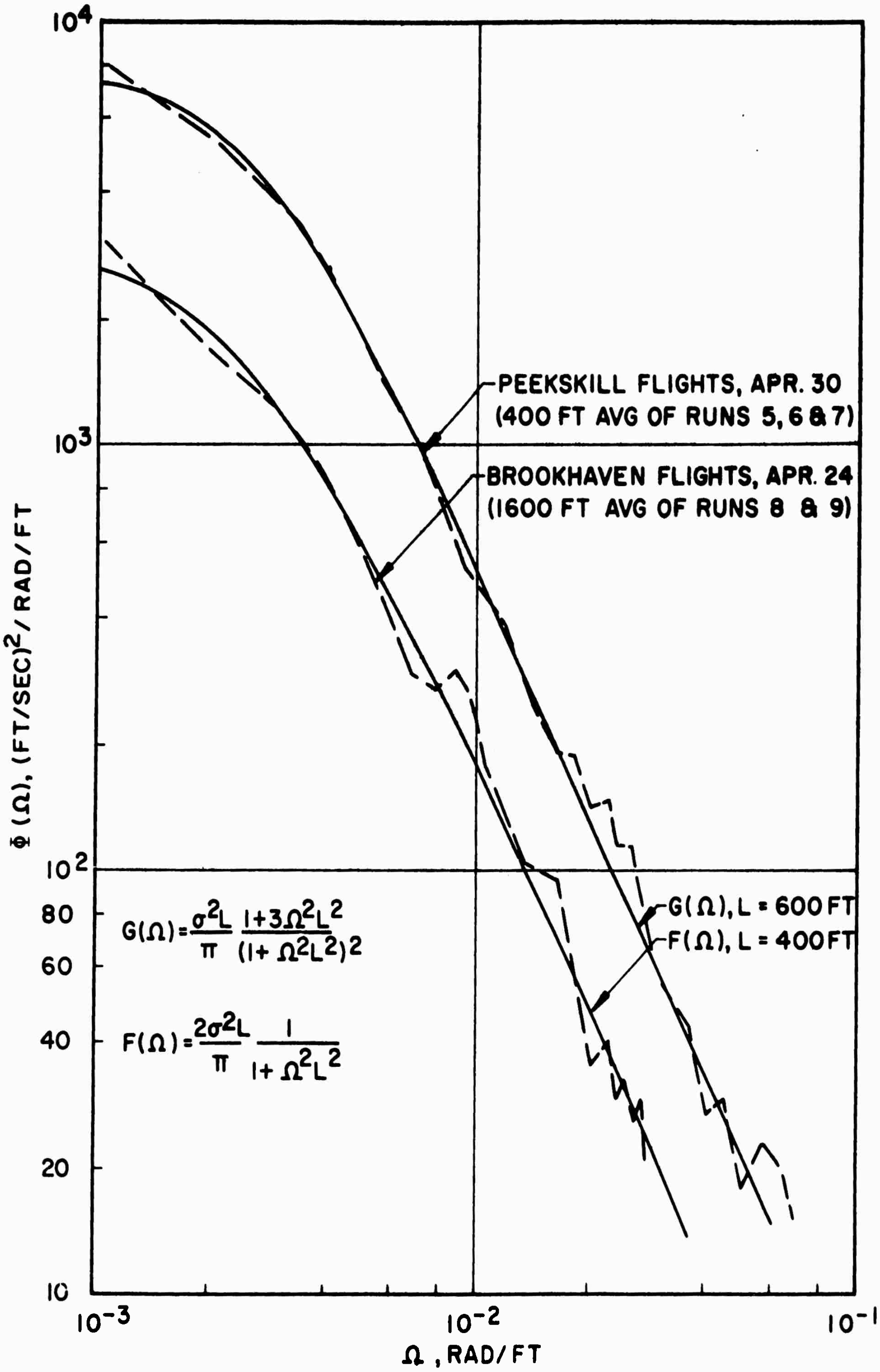


FIGURE 27

ENVELOPE OF VERTICAL VELOCITY SPECTRA AND COMPARISONS WITH EMPIRICAL TURBULENCE FUNCTIONS.